# APPLIED MECHANICS Reviews

A CRITICAL REVIEW OF THE WORLD LITERATURE IN APPLIED MECHANICS AND RELATED ENGINEERING SCIENCE

REVS. 4821-5351 VOL. 12, NO. 10 OCTOBER 1959



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*Published Monthly by THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS at Easton, Pa., and edited by Southwest Research Institute with the co-operation of Linda Hall Library.*

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*Subscription and Production Office:* **The American Society of Mechanical Engineers, 29 West 39th St., New York 18, N. Y., U. S. A.**

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APPLIED MECHANICS REVIEWS, October 1959, Vol. 12, No. 10. Published Monthly by The American Society of Mechanical Engineers at 20th and Northampton Streets, Easton, Pa., U. S. A. The editorial office is located at the Southwest Research Institute, San Antonio 6 Texas, U. S. A. Headquarters of ASME, 29 West 39th St., New York 18, N. Y., U. S. A. Price $2.50 per copy, $25.00 a year. Changes of address must be received at Society headquarters seven weeks before they are to be effective on the mailing list. Please send old as well as new address. . . . By-laws: The Society shall not be responsible for statements or opinions advanced in papers or printed in its publications (B13, Par. 4). . . . Entered as second-class matter, January 11, 1948, at the Post Office at Easton, Pa., under the Act of March 3, 1897. ©Copyrighted, 1959 by The American Society of Mechanical Engineers.

APPLIED MECHANICS REVIEWS

VOL. 12, NO. 10 MARTIN GOLAND *Editor* OCTOBER 1959

# QUALITATIVE METHODS IN GENERAL DYNAMICS AND CELESTIAL MECHANICS

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## I. GENERAL DYNAMICAL SYSTEMS

In almost all branches of mathematics a characteristic feature is a gradual progression from purely formal preliminary methods to more rigorous and qualitative methods. This general trend is implied by the fact that mathematics is not only the science of magnitude but also of order.

Consider, for example, the problem of motion of $n(\geq 3)$ bodies which attract each other according to the Newtonian law. May one not ask, whether one of the bodies will remain forever in a certain region of the sky, or whether it will recede to infinity? Or, whether the distance between any two bodies in the $n$-body problem will increase indefinitely as the time $t \to \infty$, or whether it will remain between certain limits?

These and many similar questions of a qualitative nature would be answered immediately if it were possible to trace the trajectories in the $n$-body problem.

The proof, by Lejeune Dirichlet [1], of Lagrange's theorem on stability of equilibrium of a conservative system and the proof of the converse theorem by Lyapunov [2] are also qualitative studies. The former theorem states that "Whenever in a certain equilibrium position of a holonomic scleronomic conservative system its potential energy is minimum, the equilibrium in that position is stable," and the latter states that "If the potential energy is not a minimum at the state of equilibrium, this equilibrium is unstable in the following two cases: (i) when the absence of the minimum of the potential energy is recognizable from second-order terms only without any need for the consideration of higher-order terms, and (ii) when the potential energy has a maximum which is determined by the lowest order terms present in the expansion of this function."

Finally, Liouville's theorem [3, 4] on integral invariants in statistical mechanics is another example of a qualitative study carried out about the middle of the past century.

In spite of these fruitful early attempts and the beauty of the results obtained, no further research in this line was made until the eighties of the past century. At that time Poincaré [5] undertook the theory of differential equations from a new point of view. His approach consisted in the collective, and no longer individual, consideration of the different solutions of the given equation and their mutual relations. Poincaré's work on differential equations was stimulated by his interest in certain problems of celestial mechanics and cosmogony, in particular, the long-time behavior of stellar motions and the formation of the solar system [6, 7, 8].

Beginning with Poincaré, systems of differential equations have been investigated, the right-hand sides of which do not contain the time $t$ explicitly. By means of introduction of auxiliary variables such systems can be reduced to systems of first-order differential equations of the form

$$\dot{x}_i = X_i(x_1, \ldots, x_n) \qquad (i = 1, \ldots, n), \tag{1}$$

where the $X_i$ are real functions of the variables $x_1, \ldots, x_n$ with at least sufficient conditions imposed to insure continuity and uniqueness of the solutions. A dot above $x_i$ denotes derivation with respect to $t$. Following G. D. Birkhoff [9] such systems are now called dynamical systems. The variables $x_1, \ldots, x_n$ are considered as the coordinates of an $n$-dimensional space $E_n$, and the solutions of (1) are called the motions of the dynamical system under consideration. To each possible state of our mechanical system there corresponds a uniquely determined point of the space $E_n$, which is called the image point of the given mechanical system. The whole space $E_n$ is called the phase space of our mechanical system. Although the motion of a point or the motion of a set of points in a higher dimensional space $E_n$ cannot be visualized easily, nevertheless such a geometric interpretation of the mechanical system by means of its phase space is exceedingly fruitful, and the analogy between the properties of the space $E_n$ and the Euclidean $E_3$- and $E_2$-spaces is helpful in the discussions. Finally, from the obtained picture of motion in the phase space $E_n$ we deduce the corresponding picture of motion of the mechanical system in the real three-dimensional space.

The final aim of general dynamics can then be formulated as the qualitative determination of all possible types of motions and of their interrelation.

It was Poincaré who first reduced problems of dynamics to problems in topology and solved them. Birkhoff, who took over the techniques and problems of Poincaré and carried on, was the first to undertake a systematic development of dynamics by means of topological methods. In this connection must be mentioned Birkhoff's [9, 10] fundamental researches concerning central motions, recurrent motions and the existence of periodic motions near a periodic motion.

The development of the theory of general dynamical systems possessing integral invariants, of which the Hamiltonian systems of classical dynamics is a special case, led Poincaré to his recurrence theorem [7, 11] and Birkhoff to the so-called ergodic theorem [3, 4, 10] which is of great importance in statistical mechanics.

Beginning with the thirties of the present century the Soviet mathematicians have also taken an active role in the development of the theory of general dynamical systems. At the present time this theory can be subdivided into two parts: topological theory and metric theory. Both parts of the theory have been presented rather in full in a monograph by Nemytskiĭ and Stepanov [12], and in a paper by Nemytskiĭ [13]. The topological theory alone is to be found in a recent monograph by Gottschalk and Hedlund [14] and the metric theory in two monographs by Hil'mi [15, 16], one of which is of a recent date. Nowadays these fields of study are attracting interest of more and more mathematicians.

## II. THE PROBLEM OF $n$-BODIES

Since the advent of celestial mechanics the problem of $n$-bodies has occupied a central position. With the shift of several basic problems from celestial mechanics to geophysics, its importance has only increased. Therefore we may say that the practical importance of this problem widely transcends the boundaries of its astronomical applications.

The solution of the problem of motion of $n$ gravitating particles consists in the determination of the positions and velocities of these particles in their subsequent motion, assuming that they are free to move in space and are initially moving in any given manner. This, however, is a complicated nonlinear problem which cannot be solved in finite terms by means of any of the functions at present known in analysis.

Due to some simplifying circumstances existing in the solar system it was possible to develop methods of approximation for the solution of problems of motion of planets, comets and satellites. These methods, known as perturbation methods, made extensive use of infinite series. The use of infinite series, however, requires investigation of their convergence and an estimation of the error committed by replacing an infinite series by a finite number of its first terms. Due to great mathematical difficulties involved, these two problems remained for a long time in general unsolved. By the end of the past century Poincaré showed that the series of the perturbation theories do not converge but possess the nature of the so-called asymptotic series. This means that for time intervals which increase indefinitely the application of these series would lead us to false results. If, however, the time intervals in question are short compared to the lifetime of the solar system, the agreement between the results as given by the perturbation methods and the astronomical observations is amazingly good.

In the case of planets and comets the smallness of their masses implies that their mutual attractions are very small compared to the attraction exerted by the Sun. In the problem of motion of the Moon about the Earth the situation is different from the problem of motion of the planets and comets about the Sun. The perturbing body is now the Sun, the mass of which is considerably greater than that of the central body, the Earth. However, the distance from the Moon to the Sun is approximately 400 times larger than that between the Moon and the Earth. As a consequence of this fact the perturbations due to the Sun are relatively small compared to those due to the Earth. Due to these simplifying circumstances in the solar system, the general $n$-body problem, when applied to the above-mentioned special problems, reduces to the corresponding perturbation problem. In the general $n$-body problem, however, contrary to the above perturbation problems, the masses and their mutual distances must be considered as being all of the same order of magnitude.

There are three remarkable papers by Sundman [17–19] in which it is shown that, apart from a single, very special, exceptional case when all three bodies collide, the coordinates and the time in the general three-body problem can be expressed in terms of convergent power series, which are of a quite different type than those in the perturbation theories. Unfortunately, these series give no means for obtaining qualitative information as to the possible types of motion in the general three-body problem. This is so because Sundman only proved their existence but did not actually calculate them, and at least for very large time intervals these series converge so slowly that they are valueless as a basis for numerical computations. Therefore, although theoretically the Sundman solution of the three-body problem means a considerable achievement, it has not proved to be of any practical importance up to the present time (see, for example, [33]).

Another approach toward the solution of the restricted and general three-body problem was initiated by C. Burrau (1894), T. N. Thiele (1896) and G. H. Darwin [20, 21], and continued for almost four decades (1900–1938) by E. Strömgren [22, 23] at the Copenhagen Observatory [33]. Their method is that of numerical integration of the differential equations of the problem of three-bodies. This method puts us in the position to solve the three- or the $n$-body problem provided only that the initial conditions of motion are given. After such a numerical integration has been carried out we know the motion of the bodies only within the time interval for which the calculation was made, unless the motion turns out to be periodic. In the last case the motion is valid for all time. This is the reason why the problems of the existence and the actual calculation of periodic orbits has played such an important role in the works of Lagrange, G. W. Hill [24], H. Poincaré, G. H. Darwin, F. R. Moulton [25], G. D. Birkhoff [10], E. Strömgren, and others.

## III. QUALITATIVE METHODS

The qualitative investigation of a dynamical problem usually consists in the study of the behavior of an incomplete system of parameters which are selected for the purpose of deducing certain essential properties of motion of the dynamical system under consideration. The system of parameters being incomplete for the unique description of the motion, we are impelled to restrict ourselves to the finding of some fairly general laws of motion characterized by the variation with time of these parameters. Such an approach to the problem under consideration has the advantage of being able to deduce certain important results directly from the equations of motion and from certain known integrals without the knowledge of all the integrals of these equations.

In the case of the $n$-body problem, a natural and useful system of parameters is given by the mutual distances $r_{ij}$ between the bodies. Therefore, the investigation of the general properties of these distances as the time $t$ increases indefinitely (in the positive or in the negative direction) should prove of great importance for a qualitative study of the $n$-body problem.

The first general and very simple results in this line were obtained by Jacobi (1842), by considering the energy integral $T - U = h$ and the Lagrange-Jacobi equality $\ddot{J} = 2(U + 2h)$, where $J = (1/M)\ \Sigma_{ij}\ m_i\ m_j\ r_{ij}^2\ (i \neq j)$ is the moment of inertia of the system of $n$ bodies about its center of mass, assumed to be fixed and at the origin, and $M$ is the total mass of the system.

The first theorem states that if the energy constant $h < 0$ and for $t \to +\infty\ (-\infty)$ collisions between the bodies do not occur, then for all $t > 0\ (t < 0)$ the minimum distance $r(t)$ between the $n$ bodies is bounded above.

The possibility that some of the mutual distances may increase indefinitely does not contradict this theorem. The theorem only points out that in the case $h < 0$ not all of the mutual distances can increase without bounds. Consequently for $h < 0$ and $t \to +\infty$ or $t \to -\infty$ complete disintegration of $n$ gravitating bodies is impossible.

The second theorem states that if the energy constant $h > 0$ and for $t \to +\infty\ (-\infty)$ collisions do not occur, then at least one of the mutual distances increases indefinitely.

This theorem again does not contradict the case that all of the mutual distances may become infinite as $t \to +\infty$ or as $t \to -\infty$, or as $t \to +\infty$ and $-\infty$. The theorem also does not contradict the possibility of the existence of a subsystem in which all mutual distances remain finite as $t$ approaches infinity in one or in the other direction or in both directions. Finally, the theorem does not contradict the possibility of complete disintegration of the system in one direction of the time, combined with a partial disintegration in the opposite direction of the time.

Considering the two theorems mentioned above we may say that for $h < 0$ there is no sufficient energy in the system for its complete disintegration while for $h > 0$ the energy of the system is so large that at least partial disintegration of the system is unavoidable.

Beginning with the twenties of the present century J. Chazy [26–29] started to investigate systematically the long-term behavior of motion of three gravitating bodies. He obtained several types of motion for positive $h$, $h = 0$, and negative $h$ in terms of the orders of magnitude (for large $t$) of the three mutual distances $r_{ij}$ of the three bodies. Such types of motion are: hyperbolic ($r_{ij} \sim t$), hyperbolic-parabolic ($r_{13}, r_{23} \sim t$; $r_{12} \sim t^{2/3}$), hyperbolic-elliptic ($r_{13}, r_{23} \sim t$; $r_{12}$ finite), parabolic ($r_{ij} \sim t^{2/3}$), bounded and oscillating (at least one of the $r_{ij}$ is neither infinite nor bounded).

Except for bounded and oscillating types of motion there are three classes of each type, depending on which of the three bodies recedes to infinity.

For the case $h < 0$ Chazy proved (1929) the non-existence of pencils of trajectories which are bounded as $t \to +\infty$, and hyperbolic-elliptic as $t \to -\infty$, or conversely. Later this theorem was misinterpreted as a proof for the impossibility of the capture in the general three-body problem, although this result was proved only for $h < 0$. Merman [30] recently showed that almost all motions which are hyperbolic-elliptic for $t \to +\infty$ are also of the same type for $t \to -\infty$. The question remains open as to whether these hyperbolic-elliptic motions belong to the same class or not.

For $h > 0$ Chazy's investigations (1932) left the question of capture open. Finally in 1947, O. Yu. Shmidt [31, 32] produced a numerical example, illustrating the occurrence of capture in the general three-body problem. He showed that one body can approach another from infinite distance (in the past) and then remain at bounded distance from the second body as $t \to +\infty$. Once the possibility of capture was shown, the need arose for elaboration of effective qualitative criteria, given in the form of inequalities with respect to the initial data of motion, in order that certain types of motion in the three-body problem be possible. During the years 1948–1951 this mathematical problem was attacked by G. Hil'mi [15], who gave the first sufficient criteria for the realization of hyperbolic and hyperbolic-elliptic motions in the general three-body problem. At present there is a vast literature on the capture problem (see, for example, [33]).

In what follows we shall briefly characterize three methods for the qualitative investigation of dynamical systems, including systems of $n$ gravitating bodies, and we shall mention some of the main results gained up to date by these methods. The picture so obtained of the problems is far from complete. Much more hard work and ingenious thinking will be required in the future before a satisfactory survey of the possible types of motions and their interrelation in the $n$-body problem and in dynamical systems in general will be obtained.

## 1. METHOD OF DIMENSIONAL ANALYSIS

This method, which has wide applications in various branches of mechanics and physics, has recently been applied by Hil'mi [16] to the problem of $n$-bodies. His aim was to use this method for obtaining criteria in order that certain types of motion in the system of $n$ gravitating bodies take place. The idea that dimensional analysis could lead us to the analytic form of the relations existing between certain dimensional quantities is based on the assumption that a physical connection between dimensional quantities cannot depend on the chosen system of units. On the other hand, such connections cannot be arbitrary and must be determined by the dimensions of the quantities involved. When the dimensional analysis is successful in leading us to a certain criterion, this criterion is determined up to a dimensionless factor which cannot be determined by means of dimensional analysis alone. Therefore we cannot expect to obtain final results from the application of dimensional analysis to the $n$-body problem but this method proves to be useful in the preliminary stages of investigation in putting our problem in the proper perspective.

Dimensional analysis is usually applied to mathematical relations given in the form of equalities or equations. A new feature in the application of dimensional analysis to the $n$-body problem consists in introducing inequalities.

Consider $n$ gravitating bodies $P_1, \ldots, P_n$ with masses $m_1, \ldots, m_n$. Denote as before by $r_{ij}(t)$ the distance between $P_i$ and $P_j$ and by $r'_{ij}(t)$ its derivative with respect to $t$. Let

$$r(t) = \min \{r_{ij}(t)\} \text{ and } r'(t) = \min \{r'_{ij}(t)\} \qquad (i \neq j).$$

Assume that at the initial instant $t = 0$ the inequality $r'(0) > 0$ (2) is satisfied. Then two cases are possible:

(1) there exists a positive constant $\tau$ such that for all $t$ belonging to the interval $0 \leq t < \tau$ the mutual distances between the bodies continue to increase, whereas beginning with the instant $\tau$ at least one of the distances ceases to do so; or

(2) if $r'(0)$ and consequently also the initial values of the radial velocities $r'_{ij}(0)$ are so large that the mutual attractions are insufficient to prevent the total disintegration of the system, all the mutual distances $r_{ij}(t)$ increase indefinitely as $t \to +\infty$.

The question is, for what values of $r'(0)$ will the second possibility take place? Obviously, this will depend on the values of the parameters defining the initial state of motion of the system, and on the parameters characterizing gravitational phenomena.

Two obvious parameters characterizing the state of motion of the system are $r(0)$ and $r'(0)$. Taking into account the $n$ masses $m_1, \ldots, m_n$, we obtain $n$ additional parameters. However, in order to reduce the number of parameters, we may introduce a certain function of all the masses, say, $\mu = \varphi(m_1, \ldots, m_n)$ as the third parameter. Finally, the gravitational constant $g$ must be introduced as the fourth parameter. If for the given values of $r(0)$ and $\mu$ the value of $r'(0)$ is sufficiently large, disintegration of the system of $n$ bodies will occur no matter what are the values of the other parameters of the system which have been neglected so far. Therefore we arrive at the conclusion that from purely physical considerations the criterion we are looking for must be of the form

$$r'(0) > f(g, \mu, r(0)),$$

where $f$ is an unknown function. The dimensional analysis gives to this function the expression $f(g, \mu, r(0)) = C(g\mu/r(0))^{1/2}$. Having in mind the usual system of units, we can put $g = 1$. Hence our criterion sought assumes the form

$$r'(0) > C(\mu/r(0))^{1/2}, \qquad (3)$$

where the dimensionless constant $C$ and the form of the function $\varphi$ cannot be obtained by dimensional analysis. Thus the dimensional analysis indicates only the analytic form of a sufficient criterion for a complete disintegration of $n$ gravitating bodies but it does not give the final expression of this criterion as it was pointed out before. The truth of inequality (3) is confirmed by direct analysis of the equations of motion.

## 2. METHOD OF CONTINUOUS INDUCTION

This method is due to Hil'mi [16] and is more effective than the previous one. When applied to the $n$-body problem, it gives sufficient conditions, in terms of the initial data of motion of the system, in order that certain types of long-term motion be realized.

Consider a set $E$, the elements of which are arbitrary mechanical or physical systems $S$ which change in the course of the time. Let $\pi$ be a certain property of $S$.

*Definition 1.* The property $\pi$ will be said to be inductive on the set $E$ if for every $S \in E$, from the assumption that this property holds for all $t$ for which $0 \leq t < \tau$, follows that this property also holds for $t = \tau$, no matter what is the value of $\tau$.

*Definition 2.* The property $\pi$ will be called continuous if from the assumption that this property holds at a certain instant $T$ follows that it holds also in a certain neighbourhood of the instant $T$.

*Theorem on continuous induction.* If the continuous property $\pi$ which is inductive on the set $E$ holds at the initial instant $t = 0$, then it also holds for all $t > 0$.

For the application of this theorem it is necessary to have certain criteria which assure us that a particular property of a dynamical system is inductive on a set. For some sufficiently general cases such criteria have been given by Hil'mi.

Assume that the mechanical system $S$ under consideration is characterized to some extent by a finite set of time-dependent parameters $\{\omega\}$ and a second finite set of time-independent parameters $\{a\}$. Denote by $A(\{a\}, \{\omega\})$ an operator which according to certain rules assigns to every set of values of the constant parameters $a_1, \ldots, a_\mu$ and to every set of functions $\omega_1(t), \ldots, \omega_\nu(t)$ a continuous function $F(t)$, defined for all $t \geq 0$. Further assume that the functions $\omega_\nu(t)$ are unknown but that our knowledge concerning the behavior of the system $S$ can be expressed in terms of these functions and the parameters $a_1, \ldots, a_\mu$ in the form of inequalities

$$\omega_i(t) \geq A_i(\{a\}, \{\omega\}) \quad (i = 1, \ldots, \nu), \tag{4}$$

where the expressions for the operators $A_i$ are given.

Let $f_1(t), \ldots, f_\nu(t)$ be a chosen set of continuous functions defined for all $t \geq 0$. Let the property $\pi$ consist in the fulfillment of the inequalities

$$\omega_i(t) > f_i(t) \qquad (i = 1, \ldots, \nu). \tag{5}$$

*Theorem on inductivity of inequalities.* Let the system $S$ satisfy the operator inequalities (4), and from the assumption that the inequalities (5) hold for all $0 \leq t < \tau$ ($\tau \geq 0$ and arbitrary) let follow the fulfillment of the inequalities

$$A_i(\{a\}, \{\omega\}) \geq A_i(\{a\}, \{f\}).$$

Then the property (5) is inductive on the set $E$ of systems $S$ for which the inequalities

$$A_i(\{a\}, \{f\}) - f_i(t) > 0$$

are satisfied for all $t \geq 0$.

As an example for the application of this theorem consider again the $n$-body problem, using the so-called Jacobian coordinates.

Considering the motion of an arbitrary particle $P_s$ with respect to another particle $P_k$, Hil'mi derived the inequality

$$r_{ks}(t) \geq r_{ks}(0) + r'_{ks}(0)t - (1/\mu_{ks}) \Sigma_{ij} \int_0^t \int_0^t (m_i m_j / r^3_{ij}) dt dt, \tag{6}$$

where

$$\mu_{ks} = m_k m_s / (m_k + m_s).$$

We shall use the inequality (6) for finding sufficient conditions in order that complete disintegration of the system of $n$ bodies takes place, i.e. in order that all mutual distances increase indefinitely as $t \to +\infty$.

This will certainly be the case if the condition $r_{ks}(t) > \frac{1}{2}[r_{ks}(0) + r'_{ks}(0)t]$ (7) holds for all $t \geq 0$.

Take for the functions $\omega_i(t)$ the mutual distances $r_{ks}(t)$ and for the parameters $a_j$ their initial values $r_{ks}(0)$, the initial values of their derivatives $r'_{ks}(0)$ and the masses $m_i$. For the comparison functions $f_{ks}(t)$ take the right-hand sides of (7) and for the operators $A_{ks}$ the expression on the right-hand side of inequality (6). The property $\pi$ will then consist in the assumption that the inequalities $r_{ks}(t) > f_{ks}(t)$ are satisfied.

With these notations the last theorem leads us to the following result.

*Theorem.* If at the initial instant $t = 0$ the system of $n$ gravitating bodies satisfies the conditions

$$r'_{ij}(0) > 0 \tag{8}$$

$$r'_{ij}(0) - (8/\mu_{ks}) \Sigma_{ij} m_i m_j / r_{ij}(0)\, r'_{ij}(0) > 0,$$

then the mutual distances $r_{ij}$ increase indefinitely as $t \to +\infty$.

It is easy to make sure that inequalities (8) will be satisfied provided that

$$r'(0) > 0, \; r'^2(0) > 8M^*/r(0), \tag{9}$$

where

$$M^* = \Sigma_{ij} m_i m_j / \min \mu_{ij}.$$

It is interesting to note that the conditions (9), which are sufficient for complete disintegration of $n$ gravitating bodies, are exactly of the same form as the conditions (2) and (3) obtained by the method of dimensional analysis.

## 3. METHOD OF INVARIANT MEASURE

This method studies mechanical or physical systems from the point of view of trajectories in the phase space $E_n$.

We shall assume that the conditions which guarantee the existence, uniqueness and continuity of the solutions of the dynamical system (1) are satisfied. The physical meaning of the existence and uniqueness theorems is that the motion of the dynamical system (1) is completely determined by the differential equations and the initial conditions. A consequence of these theorems is that through any point of the domain $D$ of existence of solutions there passes one and only one trajectory, so that, in particular, no trajectory can cross itself.

Assume that the initial instant is $t = 0$ and denote by $f(p, t)$ the solution of the system (1) passing through the point $p = p(x_1, \ldots, x_n)$ of $D$. Then the solution $f(p, t)$ is uniquely determined in one of the intervals $-\infty < t < +\infty$, $-\infty < t < t''$, $t' < t < +\infty$, $t' < t < t''$, where, as $t$ approaches $t'$ or $t''$, the point $f(p, t)$ approaches the boundary of the domain $D$.

The function $f(p, t)$, considered as a function of two arguments, the point $p$ and the numerical argument $t$, possesses the following properties: (i) $f(p, 0) = p$, (ii) $f(p, t)$ is continuous with respect to any set of arguments $p$, $t$ in the domain of existence of the solutions, (iii) for any $p$ of $D$ and any real $t$ and $t_1$ of the $t$-interval on which the solution exists the group property $f[f(p, t), t_1] = f(p, t + t_1)$ holds. From the properties (i) and (iii) also follows the existence of the inverse transformation $f(p, -t)$ of $f(p, t)$. Because of (i) the identical transformation of the group corresponds to $t = 0$.

Hence, if for any point $p$ in $D$ the function $f(p, t)$ is defined on the whole $t$ range, then $f(p, t)$ defines a one-parameter group of transformations of the domain $D$ into itself.

Physically speaking the dynamical system (1) defines a continuous flow in the phase space $E_n$. Many results for flows which are of interest for classical dynamics have been developed since the time of Poincaré. However, the extension of these results from flows to transformation groups is the work of recent years.

The phase space $E_n$, being a generalization of the Euclidean $E_3$- and $E_4$-spaces, is a metric space in which generalized theories of point sets and of measure of sets hold. These

theories, the elements of which we shall use, are exposed in courses of the theory of functions of a real variable, as, for example, [34].

The main objects of investigation in general dynamics are invariant point sets of the phase space $E_n$. A set $A$ is said to be positively (negatively) invariant or invariant for $t \to +\infty$ $(-\infty)$ if for every point $p \epsilon A$ the motion $f(p, t)$ is defined for all $t \geq 0$ $(t \leq 0)$ and at these instants $f(p, t) \epsilon A$. A set is called invariant if it is invariant for $t \to +\infty$ as well as for $t \to -\infty$. The set $\Sigma^+$ $(\Sigma^-)$ is defined as the set of all $p$ for which $f(p, t)$ is defined for all $t \geq 0$ $(t \leq 0)$. The intersection $\Sigma$ of the sets $\Sigma^+$ and $\Sigma^-$ is called the regular kernel of a dynamical system. Hil'mi proved that almost all points of $\Sigma^+$ give rise to trajectories which are either unstable for $t \to +\infty$ or are such that on the trajectory the potential energy of $n$ gravitating bodies is "oscillatory."

$A$ being a set of the $n$-dimensional phase space $E_n$, $f(A, t)$ denotes the set obtained from $A$ following the flow through time $t$. A set $A$ is recurrent for $t \to +\infty$ if $f(A, t)$ meets $A$ for arbitrary large $t$; a point $p$ is recurrent if $f(p, t)$ meets each $\epsilon$-neighborhood of $p$ for arbitrary large $t$.

The importance of the concept of the measure of a set is due to the fact that for motions in the phase space of a dynamical system the theorem of Liouville on the invariance of the measure holds. This property of the invariance of the measure in the phase space permits us to reveal some very important properties of dynamical systems.

*Theorem of Liouville.* Let the differential equations (1) be such that $\operatorname{div} X(X_1, \ldots, X_n) = 0$ (the special form of Hamiltonian systems, for example, satisfies this condition). Further let $A$ be any measurable (in the sense of Lebesgue) set of points of the phase space $E_n$ of the given mechanical system. In addition assume that mes $A$ is finite and that for any point $p \epsilon A$ the motion $f(p, t)$ is defined for all $t \geq 0$ $(t \leq 0)$. Then mes $f(p, t) =$ mes $A$ for all $t \geq 0$ $(t \leq 0)$.

In other words, the measure of measurable point sets is an invariant of the motion of the phase space $E_n$.

The additional assumption that for any $p \epsilon A$ the motion $f(p, t)$ must be defined for all $t \geq 0$ $(t \leq 0)$ is necessary since otherwise the set of points $f(p, t)$ and its measure may not be defined for all $t \geq 0$ $(t \leq 0)$. Such a possibility may occur even in the case of very simple differential equations. Therefore the omission of this additional assumption may lead occasionally to some false deductions.

Next let us mention a couple of recurrence theorems culminating in the classical Poincaré-Carathéodory theorem.

*Theorem of Poincaré.* Let $A \subset E_n$ be a set of finite invariant measure which is also invariant for $t \to +\infty$ $(-\infty)$. Then any subset $B$ of positive measure of $A$ is recurrent for $t \to +\infty$ $(-\infty)$.

This theorem states only the existence of arbitrarily large absolute values of $t$ for which the set $f(B, t)$ has non-zero intersection with the set $B$. The measure of the intersection and the frequency of the recurrence time, however, are not estimated.

A. Khinchin [35] showed that (i) the measure of this intersection satisfies the inequality

$$\text{mes } Bf(B, t) > \frac{(\text{mes } B)^2}{\text{mes } A} - \epsilon \quad (\epsilon > 0, \text{ arbitrary})$$

and (ii) the values of $t$ for which (i) holds have a "relatively dense" distribution on the $t$-axis, i. e. for a sufficiently large $T$ the interval $(a - T, a + T)$, no matter what is $a$, contains at least one such value of $t$. In other words, the intersection of the wandering set $f(B, t)$ with its original position $B$ has not only a positive measure but this measure has a greatest lower bound which depends only on the measures of the sets $A$ and $B$. Further, with respect to the time $t$ this intersection takes place in a relatively dense set of $t$ values.

*Theorem.* Let $A \subset E_n$ be a measurable set such that any of its subsets of positive measure is recurrent for $t \to +\infty$ $(-\infty)$. Then all points of $A$, except possibly points of a set of measure zero, are recurrent for $t \to +\infty$ $(-\infty)$.

Contrary to the previous theorem, this last one does not make the assumption that the set $A$ is invariant and that its measure in finite.

From the last two theorems immediately follows the classical Poincaré-Carathéodory theorem.

*Theorem of Poincaré and Carathéodory.* If $A \subset E_n$ is a set of finite invariant measure which is also invariant for $t \to +\infty$ $(-\infty)$, then all points of the set $A$, except possibly points of a set of measure zero, are recurrent for $t \to +\infty$ $(-\infty)$.

In a nonmathematical language the recurrence theorem of Poincaré-Carathéodory states that on almost every trajectory in $A$ the moving point in the course of time comes arbitrarily close to its initial position infinitely often. This statement is true for both directions of the time. Since any point on a trajectory may be considered to be an initial point, it follows that the moving point comes back arbitrarily close to every point on a trajectory an infinite number of times.

In other words, the recurrence theorem shows that the evolution of a mechanical system is quasi-periodic, provided only that certain conditions are fulfilled.

The property of quasi-periodicity is not possessed by all trajectories, but the exceptional trajectories without this property are in a negligibly small minority.

Finally it should be observed that due to small outside disturbances to which a real mechanical system is always subject, such a system can never be self-contained. Therefore the assumptions of the Poincaré-Carathéodory recurrence theorem are never fulfilled in reality. Nevertheless, the recurrence theorem will be applicable if the recurrence time is small compared with the time interval during which the outside disturbances may add up to forces that can influence appreciably the behavior of the mechanical system.

*Two theorems of E. Hopf* [36]. If we turn our attention from invariant sets of finite measure to the regular kernel $\Sigma$ of a dynamical system which in particular cases may coincide with the whole phase space or may be a set of infinite measure, then the Poincaré-Carathéodory recurrence theorem is no more applicable. Nevertheless, in such cases some general laws hold which give a description, in the large, of the types of motions a dynamical system with a positive integral invariant may have.

*Definition.* A point $p$ is called a point of escape with respect to the future or the past if the distances from $p$ of the points $f(p, t)$ increase beyond all bounds as $t \to +\infty$ or as $t \to -\infty$.

It is again evident that if a point on a trajectory is a recurrence point or a point of escape with respect to a particular direction of the time, then all points on that trajectory have the same property.

Hopf's first theorem, which is a generalization of the Poincaré-Carathéodory recurrence theorem, states that, except for a set of measure zero, all points in the phase space are either recurrence points or points of escape. This holds for both the future and the past.

When applied to mechanical systems this theorem implies that, in general, the evolution of a system is quasi-periodic, or the system disintegrates. A partial answer to the question, whether recurrence or disintegration will occur, is given, in the case of systems of gravitating bodies, by Jacobi's two theorems mentioned previously. While for $h > 0$ the system necessarily disintegrates, at least partially, there is no satisfactory criterion for $h < 0$ to decide between quasi-periodicity and disintegration of the system.

If, however, it is known that the system does not disintegrate, the so-called virial theorem can be deduced from the Lagrange-Jacobi equality. This theorem states that both the

kinetic and potential energies have time averages which are simple multiples of the total energy $h$.

Hopf's second theorem, which connects the descriptions of trajectories with respect to the past and the future, states that, except for a set of measure zero, all initial points which are points of escape or recurrence points with respect to the past are, respectively, points of escape or recurrence points with respect to the future. In other words, the types of motion are "almost always" the same for both the past and the future. A priori we could expect this result since neither direction of the time is preferred by Newton's equations of motion.

For a number of generalizations of the concepts and theorems mentioned in this article and their applications to cosmogony and stellar dynamics the reader is referred to the literature, for example, [15, 16, 37].

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# Analytical Methods in Applied Mechanics

(See also Revs. 4830, 4854, 4862, 4863, 4866, 4881, 4888, 4894, 4954, 5093, 5106, 5124, 5189, 5193, 5276, 5294, 5321)

**4821. Roth, J. P., An application of algebraic topology: Kron's method of tearing,** *Quart. Appl. Math.* **17**, 1, 1–24, Apr. 1959.

Paper reviews history of electrical network problem, including Kron's methods and author's algebraic topology proofs regarding existence of solutions of Kron's problem and invariance of power under tearing. (Reviewer believes last does not adequately discuss all difficulties.) Kron's method of solving by tearing and recombining is explained and clarified. Author considers in detail three particular networks and compares numbers of multiplications needed to solve them by various methods. Typical result: Standard inversion methods, 4096 multiplications; standard partitioning, 1697; modified partitioning based on certain Kron ideas, 618; Kron's tearing method, 368. Emphasized further is important advantage of tearing over other methods, namely that since component solutions represent actual physical systems, they can be combined into ever larger systems. Reviewer regards paper as a significant contribution.

B. Hoffmann, USA

**4822. Bell, P. O., Some applications of differential geometry to the interpretation of physical phenomena,** *Quart. Appl. Math.* **16**, 4, 321–333, Jan. 1959.

Author states and proves several theorems about functions in an $n$-dimensional Euclidean space which are generalizations of similar theorems about heat conduction and electric charge distribution in three dimensions. One theorem gives the relation between the Laplacians in $n$ space and in the $n-1$ hypersurface contained in the $n$ space. Other theorems correlate the generalized heat flow rates with the curvatures of the "surfaces" and "trajectories" or, alternatively, give the "charge" density. The final two theorems are proven in three dimensions but expressed in a form valid in $n$ dimensions.

An interesting but abstract discussion of the differential geometric properties of harmonic and related functions.

R. E. Street, USA

**Book—4823. Noble, B., Methods based on the Wiener-Hopf technique,** New York, Pergamon Press, 1958, x + 246 pp. $10.

Book under review is meant to acquaint the student with a powerful technique which is useful in the solution of a large class of boundary-value problems in partial differential equations. The technique is that due to Wiener-Hopf and the examples solved by it are culled from the fields of electromagnetic theory, acoustics, hydrodynamics, elasticity and potential theory. From the topics listed one can surmise the versatility of the method.

Understanding of the methods based on the Wiener-Hopf technique requires that the reader be familiar with transform calculus (Laplace, Fourier or Mellin), the corresponding complex inversion formulas, and contour integration. Some supplemental material in Chapter I on complex variables dealing with branch points, branch cuts and analytic continuation serves to make the transition to succeeding chapters smoother and easier than it otherwise would be.

Successive chapters deal with half-plane problems, further wave problems, extensions and limitations of the method, and some approximate methods. Each chapter is followed by an excellent set of problems. The book ends with a fairly extensive bibliography.

Reviewer would have preferred to see the author settle on the Laplace transform, as perhaps more widely used, rather than the Fourier transform. However, this is a minor matter as it is easy to pass from one to the other. In the opinion of the reviewer, this book should be in the hands of everyone interested in wave propagation phenomena. It brings together in compact and lucid form material hitherto scattered throughout the mathematical literature. Furthermore, a close reading should suggest further problems for research. E. J. Scott, USA

**4824. Garabedian, P. R., Application to supersonic flow; Cauchy's problem of the elliptical differential equation** (in Italian), *Atti Accad. Naz. Lincei, R. C. Cl. Sci. Fis. Mat. Nat.* (8) **24,** 3, 282–286, Mar. 1958.

Author has previously described [AMR **11** (1958), Rev. 2711] a scheme for numerical construction of a plane compressible flow with a prescribed detached shock wave. Roughly speaking, it depends on interpreting one of the independent variables as a complex variable and thereby replacing the elliptic equation for the stream function by a hyperbolic equation. In this paper the basic ideas for this procedure are developed for the simpler equation $\Delta\psi = \psi^2$ with initial data on $x = 0$ or on $x^2 - y^2 = 1$. Discussion of the latter initial curve suggests that from the form of the initial curve and initial data one can determine qualitatively the location and nature of the singularities of $\psi$. J. H. Giese, USA

**4825. Hart, V. G., The hypercircle method applied to the solution of the biharmonic equation, Part I,** AFOSR TN 58-172 (ASTIA AD 152 203), 62 pp., Dec. 1957.

The hypercircle method of Prager and Synge [AMR **11** (1958), Rev. 733] is applied to some physical problems governed by the biharmonic equation. The application to the slow flow of a viscous fluid and the bending of thin plates is discussed. The method is applied to find upper and lower bounds to the strain energy of a square clamped plate. Estimates of the central deflection and bending moments are given. Author uses Synge's technique of approximation by means of "patch functions." Problem was finally reduced to the solution of a number of simultaneous linear equations. R. C. Roberts, USA

**4826. Cicala, P., Problems of optimization with discontinuous solutions** (in Italian), *Monogr. Lab. Aero. Politecn. Torino* **91,** 416, 27 pp., 1956–57.

The first part of this paper extends the theory of the calculus of variations to problems involving discontinuous solutions of the classical problem of this calculus that requires that a certain integral have a stationary value. This first part is a general discussion of problems of this type. The remainder of the paper is concerned with definite applications to problems involving discontinuities in levels in hydraulic problems, discontinuities in demand in commercial problems involving variations of production, the classical "gold-mining" problem is considered. The paper concludes with a discussion of a torsional oscillation problem involving structures that have minimum weight.

In reviewer's opinion, this paper is a valuable contribution to the theory and application of variational problems whose solutions involve discontinuous functions and should prove of value and interest to applied mathematicians working on dynamic problems and problems in operations research.

L. A. Pipes, USA

**4827. Celestre, P., Generalized theory of dimensional analysis and similitude** (in English), 9th Congrès Intern. Mécan. Appl., Univ. Bruxelles, 1957; **2,** 157–164.

After considerable allowance for interpretation of the English language in this paper reviewer finds title is misleading (no generalized theory is developed), review of literature is not current (last dated reference is 1940), and Buckingham's Pi theorem is not adequately stated. Because proposed extensions of Pi theorem are either trivial or mathematically inconsistent with assumptions of Pi theorem, reviewer agrees with author that suggested extensions are "not above criticism." Although author indicates that "help of mathematicians would be profitable" in developing a "real Theory of Physical Functions," author seems unaware of developments given by G. Birkhoff [AMR **3** (1950), Rev. 2692], H. Huntley [AMR **6** (1953), Rev. 1459], and H. Langhaar [AMR **4** (1951), Rev. 4042]. Since publication of the author's paper, L. Brand has given a careful exposition of underlying assumptions of dimensional analysis, and established a new standard for mathematical clarity and elegance in derivation of the Pi theorem [Cf. *Arch. Rational Mech. Analysis* **1,** no. 1, 1957]. Sound mathematical efforts to generalize the theory of dimensional analysis may stem from the firm foundation given by Brand. R. E. Thomas, USA

**Book—4828. Hix, C. F., Jr., and Alley, R. P., Physical laws and effects,** New York, John Wiley & Sons, Inc., 1958, 291 pp., $8.75.

This is essentially a catalogue of physical phenomena from all fields of science, and is intended as a reference source when less common physical effects are encountered. The main portion of the book consists of an alphabetical list of the phenomena mentioned with (I) brief description, (II) illustration of some observable result, (III) indication of order of magnitude, (IV) references to more detailed discussions of the topic. The physical quantities involved in the law are finally listed and these are cross referenced at the back: there is also a cross reference by fields of science. Certain phenomena not mentioned earlier are briefly discussed in an appendix.

Unfortunately, the restrictions on the uses of the laws are not always adequately stated and this is a serious defect in a reference book. Many important matters are discussed only in the appendix, while units in the physical equations are often inadequately discussed. Specialists in any field will find that there are defects in the discussion, especially of general effects such as fluid flow or stress-strain relations. The principal benefit to be derived from this book would appear to be the collection together of many basic physical effects from the general field of science, thus providing a link between the worker in one field and the phenomena of another; it should be noted, however, that in no sense is it a complete compilation of the laws of physics and chemistry.

S. R. Montgomery, England

**Book—4829. White, D. C., and Woodson, H. H., Electromechanical energy conversion,** New York, John Wiley & Sons, Inc., 1959, xv + 646 pp. $12.50.

The interaction of electromagnetic fields and material bodies in motion will result in electromechanical energy conversion. (Electrical Machines). This text-book gives a highly analytical and generalized treatment of these interactions. Systems with mechanical and electrical interactions are treated by using the Lagrangian state function and the Legendre transformation. Mathematical technics shown for the solution of transducer problems include differential equations with constant coefficients and Laplace transforms. The solution of nonlinear problems are described by using analog techniques. The rotating machine is treated as an ideal one, consisting of magnetic cylinders and current sheets. Complete dynamic equations of machine are obtained. There is an introduction to control system theory and there are several chapters treating special devices in detail.

P. P. Biringer, Canada

## Computing Methods and Computers

(See also Revs. 4854, 4925)

**Book—4830. Kantorovich, L. V., and Krylov, V. I., Approximate methods of higher analysis** (Translated from the 3rd Russian edition by C. D. Benster), New York, Interscience Publishers, Inc., 1958, xii + 681 pp. $17.

As is well known, most mathematical problems do not admit of simple solutions in closed form. More and more we rely on approximate solutions which are sufficiently accurate for the problem at hand. To obtain them we resort to approximate methods of analysis which may be numerical or analytical in nature.

The admirably translated treatise under review should be of value to the pure or applied mathematician, the research or practical engineer, and to the physicist and chemist. The book is a source of ideas for further research as well as a compendium of useful methods that can be applied to many problems that arise in science and engineering. The scope of the work can be judged partially by the following topics that are treated: Fourier series, infinite systems of equations, boundary-value problems, double series, integral equations of the Fredholm type, difference equations, variational methods, method of Ritz, error analysis, conformal mapping, Green's functions, Dirichlet problem, Neumann problem, harmonic and biharmonic problems and the method of successive approximations.

Most of the topics mentioned are treated adequately, sometimes fully. One would like to have seen a more complete treatment of Green's functions and applications of the net method to hyperbolic and parabolic partial differential equations. However, these are minor matters which in no way detract from the excellence of this treatise of 670 pages. Although there are illustrated problems in the book, the value of the work as a text would have been enhanced if there were a collection of problems for the reader to solve at the end of each chapter.

Reviewer considers this reference an important addition to works on approximate methods of analysis.

E. J. Scott, USA

**Book—4831. Modern computing methods,** New York, Philosophical Library, 1958, vi + 129 pp. $8.75.

See AMR **11** (1958), Rev. 2923.

**4832. Sugai, I., Numerical solution of Laplace's equation, given Cauchy conditions,** *IBM J. Res. Devel.* **3,** 2, 187–188 (Short communication), Apr. 1959.

**4833. Greenspan, D., On the numerical solution of Dirichlet problems,** *Quart. J. Mech. Appl. Math.* **12,** 1, 117–123, Feb. 1959.

The results of an earlier note [AMR **10** (1957), Rev. 3160] and related facts are extended to the case in which the finite-difference grid is rectangular rather than square.

F. L. Alt, USA

**4834. Greenspan, D., On "best" nine-point Laplace difference analogues on rectangular grids,** *Quart. J. Mech. Appl. Math.* **12,** 1, 111–116, Feb. 1959.

**4835. Dickinson, J. R., A computer program for system optimization,** *Trans. Engng. Inst. Canada* **2,** 4, 157–161, Dec. 1958.

The digital computer is now used in many ways to assist the engineer in designing complex systems or structures. The formidable task of optimizing a design has, however, been infrequently tackled using a digital computer. It is possible to program the computer to optimize the design by a method which adjusts a set of independent design variables to minimize some important criterion such as the cost of the system or the weight of the structure. Some encouraging earlier experience with two optimizing programs for nuclear power reactors sparked the development of a general optimizing routine for the IBM 650 computer which can be used for a wide range of engineering problems. The program optimizes nonlinear functions of up to six independent variables and can restrain these variables within upper and lower limits. The optimizing method is a refined gradient technique which differs principally from previously published methods in using an efficient extrapolation technique to accelerate the process of solution.

From author's summary

**4836. Michalec, G. W., Survey and evaluation: analog computing mechanisms,** *Mach. Design* **31,** 6, 157–179, Mar. 1959.

A comprehensive design guide to special and general-purpose mechanisms for performing mathematical operations in all-mechanical or electromechanical computers: Addition, integration, trigonometric functions, multiplication, computing cams and gears, linkages, error analysis, system synthesis.

From author's summary

## Analogies

(See Revs. 4836, 5074, 5195)

## Kinematics, Rigid Dynamics and Oscillations

(See also Revs. 4829, 4862, 4863, 4898, 5272, 5277)

**Book—4837. Langhaar, H. L., and Boresi, A. P., Engineering mechanics—statics and dynamics,** New York, McGraw-Hill Book Co., Inc., 1959, xiv + 705 pp. $9.

This textbook is intended for sophomore and junior engineering students and presents a text which may be considered as conventional in undergraduate engineering education, but with more material than can be covered properly in one course. The material is printed in two types of print for eventual selection of topics. Contrary to the known American textbook by Seely-Ensign-Jones, "Analytical mechanics for engineers" [AMR **12** (1959), Rev. 1688], vector analysis is used, especially in the three-dimensional problems.

The first seven of the fifteen chapters are devoted to the traditional material on statics (concurrent forces, statics of rigid body, engineering applications with graphostatic problems, friction, general three-dimensional theory), giving some concepts of strength of materials and hydrostatics (distributed forces, stress,

**Book—4823. Noble, B., Methods based on the Wiener-Hopf technique,** New York, Pergamon Press, 1958, x + 246 pp. $10.

Book under review is meant to acquaint the student with a powerful technique which is useful in the solution of a large class of boundary-value problems in partial differential equations. The technique is that due to Wiener-Hopf and the examples solved by it are culled from the fields of electromagnetic theory, acoustics, hydrodynamics, elasticity and potential theory. From the topics listed one can surmise the versatility of the method.

Understanding of the methods based on the Wiener-Hopf technique requires that the reader be familiar with transform calculus (Laplace, Fourier or Mellin), the corresponding complex inversion formulas, and contour integration. Some supplemental material in Chapter I on complex variables dealing with branch points, branch cuts and analytic continuation serves to make the transition to succeeding chapters smoother and easier than it otherwise would be.

Successive chapters deal with half-plane problems, further wave problems, extensions and limitations of the method, and some approximate methods. Each chapter is followed by an excellent set of problems. The book ends with a fairly extensive bibliography.

Reviewer would have preferred to see the author settle on the Laplace transform, as perhaps more widely used, rather than the Fourier transform. However, this is a minor matter as it is easy to pass from one to the other. In the opinion of the reviewer, this book should be in the hands of everyone interested in wave propagation phenomena. It brings together in compact and lucid form material hitherto scattered throughout the mathematical literature. Furthermore, a close reading should suggest further problems for research. E. J. Scott, USA

**4824. Garabedian, P. R., Application to supersonic flow; Cauchy's problem of the elliptical differential equation** (in Italian), *Atti Accad. Naz. Lincei, R. C. Cl. Sci. Fis. Mat. Nat.* (8) **24,** 3, 282–286, Mar. 1958.

Author has previously described [AMR **11** (1958), Rev. 2711] a scheme for numerical construction of a plane compressible flow with a prescribed detached shock wave. Roughly speaking, it depends on interpreting one of the independent variables as a complex variable and thereby replacing the elliptic equation for the stream function by a hyperbolic equation. In this paper the basic ideas for this procedure are developed for the simpler equation $\Delta\psi = \psi^2$ with initial data on $x = 0$ or on $x^2 - y^3 = 1$. Discussion of the latter initial curve suggests that from the form of the initial curve and initial data one can determine qualitatively the location and nature of the singularities of $\psi$. J. H. Giese, USA

**4825. Hart, V. G., The hypercircle method applied to the solution of the biharmonic equation, Part I,** AFOSR TN 58-172 (ASTIA AD 152 203), 62 pp., Dec. 1957.

The hypercircle method of Prager and Synge [AMR **11** (1958), Rev. 733] is applied to some physical problems governed by the biharmonic equation. The application to the slow flow of a viscous fluid and the bending of thin plates is discussed. The method is applied to find upper and lower bounds to the strain energy of a square clamped plate. Estimates of the central deflection and bending moments are given. Author uses Synge's technique of approximation by means of "patch functions." Problem was finally reduced to the solution of a number of simultaneous linear equations. R. C. Roberts, USA

**4826. Cicala, P., Problems of optimization with discontinuous solutions** (in Italian), *Monogr. Lab. Aero. Politecn. Torino* **91,** 416, 27 pp., 1956–57.

The first part of this paper extends the theory of the calculus of variations to problems involving discontinuous solutions of the classical problem of this calculus that requires that a certain integral have a stationary value. This first part is a general discussion of problems of this type. The remainder of the paper is concerned with definite applications to problems involving discontinuities in levels in hydraulic problems, discontinuities in demand in commercial problems involving variations of production, the classical "gold-mining" problem is considered. The paper concludes with a discussion of a torsional oscillation problem involving structures that have minimum weight.

In reviewer's opinion, this paper is a valuable contribution to the theory and application of variational problems whose solutions involve discontinuous functions and should prove of value and interest to applied mathematicians working on dynamic problems and problems in operations research.

L. A. Pipes, USA

**4827. Celestre, P., Generalized theory of dimensional analysis and similitude** (in English), 9th Congrès Intern. Mécan. Appl., Univ. Bruxelles, 1957; **2,** 157–164.

After considerable allowance for interpretation of the English language in this paper reviewer finds title is misleading (no generalized theory is developed), review of literature is not current (last dated reference is 1940), and Buckingham's Pi theorem is not adequately stated. Because proposed extensions of Pi theorem are either trivial or mathematically inconsistent with assumptions of Pi theorem, reviewer agrees with author that suggested extensions are "not above criticism." Although author indicates that "help of mathematicians would be profitable" in developing a "real Theory of Physical Functions," author seems unaware of developments given by G. Birkhoff [AMR **3** (1950), Rev. 2692], H. Huntley [AMR **6** (1953), Rev. 1459], and H. Langhaar [AMR **4** (1951), Rev. 4042]. Since publication of the author's paper, L. Brand has given a careful exposition of underlying assumptions of dimensional analysis, and established a new standard for mathematical clarity and elegance in derivation of the Pi theorem [Cf. *Arch. Rational Mech. Analysis* **1,** no. 1, 1957]. Sound mathematical efforts to generalize the theory of dimensional analysis may stem from the firm foundation given by Brand. R. E. Thomas, USA

**Book—4828. Hix, C. F., Jr., and Alley, R. P., Physical laws and effects,** New York, John Wiley & Sons, Inc., 1958, 291 pp., $8.75.

This is essentially a catalogue of physical phenomena from all fields of science, and is intended as a reference source when less common physical effects are encountered. The main portion of the book consists of an alphabetical list of the phenomena mentioned with (I) brief description, (II) illustration of some observable result, (III) indication of order of magnitude, (IV) references to more detailed discussions of the topic. The physical quantities involved in the law are finally listed and these are cross referenced at the back: there is also a cross reference by fields of science. Certain phenomena not mentioned earlier are briefly discussed in an appendix.

Unfortunately, the restrictions on the uses of the laws are not always adequately stated and this is a serious defect in a reference book. Many important matters are discussed only in the appendix, while units in the physical equations are often inadequately discussed. Specialists in any field will find that there are defects in the discussion, especially of general effects such as fluid flow or stress-strain relations. The principal benefit to be derived from this book would appear to be the collection together of many basic physical effects from the general field of science, thus providing a link between the worker in one field and the phenomena of another; it should be noted, however, that in no sense is it a complete compilation of the laws of physics and chemistry.

S. R. Montgomery, England

**Book—4829. White, D. C., and Woodson, H. H., Electromechanical energy conversion,** New York, John Wiley & Sons, Inc., 1959, xv + 646 pp. $12.50.

The interaction of electromagnetic fields and material bodies in motion will result in electromechanical energy conversion. (Electrical Machines). This text-book gives a highly analytical and generalized treatment of these interactions. Systems with mechanical and electrical interactions are treated by using the Lagrangian state function and the Legendre transformation. Mathematical technics shown for the solution of transducer problems include differential equations with constant coefficients and Laplace transforms. The solution of nonlinear problems are described by using analog techniques. The rotating machine is treated as an ideal one, consisting of magnetic cylinders and current sheets. Complete dynamic equations of machine are obtained. There is an introduction to control system theory and there are several chapters treating special devices in detail.

P. P. Biringer, Canada

## Computing Methods and Computers

(See also Revs. 4854, 4925)

**Book—4830. Kantorovich, L. V., and Krylov, V. I., Approximate methods of higher analysis** (Translated from the 3rd Russian edition by C. D. Benster), New York, Interscience Publishers, Inc., 1958, xii + 681 pp. $17.

As is well known, most mathematical problems do not admit of simple solutions in closed form. More and more we rely on approximate solutions which are sufficiently accurate for the problem at hand. To obtain them we resort to approximate methods of analysis which may be numerical or analytical in nature.

The admirably translated treatise under review should be of value to the pure or applied mathematician, the research or practical engineer, and to the physicist and chemist. The book is a source of ideas for further research as well as a compendium of useful methods that can be applied to many problems that arise in science and engineering. The scope of the work can be judged partially by the following topics that are treated: Fourier series, infinite systems of equations, boundary-value problems, double series, integral equations of the Fredholm type, difference equations, variational methods, method of Ritz, error analysis, conformal mapping, Green's functions, Dirichlet problem, Neumann problem, harmonic and biharmonic problems and the method of successive approximations.

Most of the topics mentioned are treated adequately, sometimes fully. One would like to have seen a more complete treatment of Green's functions and applications of the net method to hyperbolic and parabolic partial differential equations. However, these are minor matters which in no way detract from the excellence of this treatise of 670 pages. Although there are illustrated problems in the book, the value of the work as a text would have been enhanced if there were a collection of problems for the reader to solve at the end of each chapter.

Reviewer considers this reference an important addition to works on approximate methods of analysis.

E. J. Scott, USA

**Book—4831. Modern computing methods,** New York, Philosophical Library, 1958, vi + 129 pp. $8.75.

See AMR **11** (1958), Rev. 2923.

**4832. Sugai, I., Numerical solution of Laplace's equation, given Cauchy conditions,** *IBM J. Res. Devel.* **3,** 2, 187–188 (Short communication), Apr. 1959.

**4833. Greenspan, D., On the numerical solution of Dirichlet problems,** *Quart. J. Mech. Appl. Math.* **12,** 1, 117–123, Feb. 1959.

The results of an earlier note [AMR **10** (1957), Rev. 3160] and related facts are extended to the case in which the finite-difference grid is rectangular rather than square.

F. L. Alt, USA

**4834. Greenspan, D., On "best" nine-point Laplace difference analogues on rectangular grids,** *Quart. J. Mech. Appl. Math.* **12,** 1, 111–116, Feb. 1959.

**4835. Dickinson, J. R., A computer program for system optimization,** *Trans. Engng. Inst. Canada* **2,** 4, 157–161, Dec. 1958.

The digital computer is now used in many ways to assist the engineer in designing complex systems or structures. The formidable task of optimizing a design has, however, been infrequently tackled using a digital computer. It is possible to program the computer to optimize the design by a method which adjusts a set of independent design variables to minimize some important criterion such as the cost of the system or the weight of the structure. Some encouraging earlier experience with two optimizing programs for nuclear power reactors sparked the development of a general optimizing routine for the IBM 650 computer which can be used for a wide range of engineering problems. The program optimizes nonlinear functions of up to six independent variables and can restrain these variables within upper and lower limits. The optimizing method is a refined gradient technique which differs principally from previously published methods in using an efficient extrapolation technique to accelerate the process of solution.

From author's summary

**4836. Michalec, G. W., Survey and evaluation: analog computing mechanisms,** *Mach. Design* **31,** 6, 157–179, Mar. 1959.

A comprehensive design guide to special and general-purpose mechanisms for performing mathematical operations in all-mechanical or electromechanical computers: Addition, integration, trigonometric functions, multiplication, computing cams and gears, linkages, error analysis, system synthesis.

From author's summary

## Analogies

(See Revs. 4836, 5074, 5195)

## Kinematics, Rigid Dynamics and Oscillations

(See also Revs. 4829, 4862, 4863, 4898, 5272, 5277)

**Book—4837. Langhaar, H. L., and Boresi, A. P., Engineering mechanics—statics and dynamics,** New York, McGraw-Hill Book Co., Inc., 1959, xiv + 705 pp. $9.

This textbook is intended for sophomore and junior engineering students and presents a text which may be considered as conventional in undergraduate engineering education, but with more material than can be covered properly in one course. The material is printed in two types of print for eventual selection of topics. Contrary to the known American textbook by Seely-Ensign-Jones, "Analytical mechanics for engineers" [AMR **12** (1959), Rev. 1688], vector analysis is used, especially in the three-dimensional problems.

The first seven of the fifteen chapters are devoted to the traditional material on statics (concurrent forces, statics of rigid body, engineering applications with graphostatic problems, friction, general three-dimensional theory), giving some concepts of strength of materials and hydrostatics (distributed forces, stress,

pressure, centers of gravity, pressure and buoyancy, etc.). The energy principles in statics are treated in detail (work, principle of virtual work, kinds of equilibrium positions, potential energy and the theorems regarding their extremal property). The next two chapters take up the kinematics and dynamics of a particle (kinematic quantities and their vector interpretations, straightline and curvilinear motion, Newton's laws of motion, dimensions and units, gravity and inertial forces, planetary motion). Chapter 10 deals with momentum principles (theorem of momentum, the law of momentum, collision of two balls, rocket flight in empty space, the applications in fluid mechanics—force of a jet, turbine blades, etc.). In the next two chapters are treated kinematics and dynamics of rigid bodies, especially translation, plane motion and rotation about a fixed axis with the theory of moments of inertia. In the next chapter the concepts of work and energy are applied to dynamical problems, e.g. flow on machines. Chapter 14 is devoted to the fundamentals of the theory of vibrations (free and forced). Several elementary technical problems are solved (e.g. vibration of a gear and of valve mechanism). The last chapter presents the introductory treatment of kinematics and dynamics in three-dimensional space by means of vector analysis. General theories of relative motion and moments of inertia are given. The gyroscopic problem is treated also, but regretfully without elementary technical applications.

Four appendices on algebraic equations, determinants, tables of centroids, and moments of inertia of solids are added. There is also an index.

There are many illustrative examples worked out throughout the text and, in addition to this, each chapter concludes with fine problems for exercises and review questions.

No original methods are used, but the treatment is very clear and on a level of students' knowledge of mathematics. Use of the conventional notations for forces, work, energies, moments of inertia is not consistent.

The book is printed carefully on good paper with excellent drawn figures, and can be warmly recommended to students, engineers and pedagogues in the field of mechanics and applied mathematics.

D. Raskovic, Yugoslavia

**Book—4838. Meriam, J. L., Mechanics, Part II—Dynamics,** 2nd ed., New York, John Wiley & Sons, Inc., 1959, xiv + 420 pp. $5.

Book is one in a long line of similar texts starting with that by Seeley and Ensign [AMR **12** (1959), Rev. 1688]. The theory is interspersed amongst approximately 700 problems. There are hundreds of pictorial illustrations, many of them overelaborate. The purpose is perhaps the psychologically questionable one of making the material attractive (palatable) to engineering students. An appendix is given on Vector Analysis, which is NOT used in the text. A good student will probably wonder about this fact. The author undoubtedly put a lot of work into the book and intends it for a large group of beginners. However, the reviewer is strongly of the opinion that the time is long past when technical education can afford the approach exemplified by the text.

W. H. Hoppmann, II, USA

**4839. Boldinskii, G. I., and Zeltin, A. I., Problem of the movement of a system of variable mass** (in Russian), *Trudi In-ta Matem. i Mekhan. Akad. Nauk Uzber SSR* no. 15, 93–98, 1955; *Ref. Zh. Mekh.*, no. 3, 1958, Rev. 2570.

A study is made of the equation of the motion of a system of variable mass, changing in consequence of the momentary union and separation of particles, recorded in the form of Lagrange equations of the second order

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_j} - \frac{\partial T}{\partial q_j} = Q_j + \frac{\partial \Pi_1}{\partial \dot{q}_j} + \frac{\partial \Pi_2}{\partial \dot{q}_j} \qquad [1]$$

where

$$\Pi_1 = \sum_{i=1}^{3n} \lambda_{1i}\dot{m}_{1i}\frac{\dot{x}_i^2}{2},\quad u_{1i} = \lambda_{1i}v_i$$

$$\Pi_2 = \sum_{i=1}^{3n} \lambda_{2i}\dot{m}_{2i}\frac{\dot{x}_i^2}{2},\quad u_{2i} = \lambda_{2i}v_i$$

The following cases of motion were examined:

1. Changing masses $m_{1i}$, $m_{2i}$ and their absolute velocities $u_{1i}$, $u_{2i}$, clearly do not depend on $q_j$ and $\dot{q}_j$.
2. Changing masses $m_{1i}$ and $m_{2i}$ depend only on $q_j$.
3. Masses $m_{1i}$ and $m_{2i}$ depend only on $\dot{q}_j$.
4. Masses $m_{1i}$ and $m_{2i}$ depend clearly on $t$ and $\dot{q}_j$.

A. I. Zenkin

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**Book—4840. McCuskey, S. W., Introduction to advanced dynamics,** Reading, Mass., Addison-Wesley Publishing Co., Inc., 1959, viii + 263 pp. $8.50.

This book is designed to correspond to a one-semester course in dynamics at the undergraduate level for students who wish to later attend graduate courses in theoretical physics. It thus attempts to bridge the gap between elementary mechanics and quantum mechanics. The first chapter treats Newton's laws and rotational motion. Hamilton's principle and Lagrange's equations are introduced in the second chapter, and rigid dynamics, oscillatory motion and motion under central forces are discussed later in the book. Basic principles are emphasized and many engineering applications which are normally covered in treatises on advanced dynamics are not touched on at all. As an introduction to the subject from a mathematical and physical point of view, however, the book fulfils its object admirably.

H. Kolsky, England

**Book—4841. Szabo, I., Introduction to technical mechanics [Einführung in die Technische Mechanik, dritte verbesserte und erweiterte Auflage].,** 3rd ed., Berlin, Springer-Verlag, 1958, xii + 419 pp. DM 22.50.

The material contained in this volume is taught to all engineering students in the first two semesters at the Technical University of Berlin. It is a thorough and comprehensive treatment of statics and dynamics in solid and fluid mechanics. To cover such broad subject in one year's time is a remarkable educational achievement, which is made feasible only by the use of such a superior text as this one.

The subject matter of the book is divided into four principal chapters. The first one (75 pages) deals with the statics and dynamics of rigid bodies. A concise and self-contained account is presented at the outset of the elements of vector calculus and, from then on, vectorial treatments are given preference in the discussions through the rest of the book. The second chapter (65 pages) contains an introduction to stress analysis. Here the author circumvents entirely the traditional restrictive approach made in elementary strength of materials (which would have to be modified or discarded later on), and starts with the basic equations of the theory of elasticity. Nothing is sacrificed, however, by this generality of approach, and in this relatively short chapter the discussion goes so far as to include such topics as beams on elastic foundation, center of shear, and a variety of torsional problems. After the short third chapter (18 pages) dealing with the statics of composite systems, the last chapter (that is practically the second half of the book) is devoted to dynamics. The wide scope of this chapter may be best illustrated by the titles of its eight subdivisions, which are: kinematics, Newton's law and its consequences, the kinematics of the gravity center, sources of resistance to motion (friction), simple vibration problems, impact,

an outline of hydromechanics, and the fundamental laws of similitude.

Among the many virtues of this book two may be mentioned in particular, since they are of unusual pedagogical value. The first one is the author's inclusion of historical references in technical discussions, chiefly because he found that such comments are apt to arouse the students' interest and participation. The second feature is the lucid but intentionally terse style of presentation, thereby expecting and prompting the student to do his own share of thinking, which is by far the greatest educational service. The best evidence for the wide and well-deserved acceptance of this book is the fact that it reached three editions in less than five years. This last edition differs from the preceding one only by the addition of an appendix containing 15 representative problems with their solutions.

M. Hetenyi, USA

**Book—4842. Szabo, I., Introduction to technical mechanics, [Einführung in die Technische Mechanik, vierte erweiterte Auflage]**, 4th ed., Berlin, Springer-Verlag, 1959, xii + 434 pp. DM 22.50.

This latest edition is enhanced by a complementary chapter on the work-equations of mechanics (principles of virtual work and of D'Alembert). The purpose of this addition was to provide an introduction to this subject for those students who might not continue their studies with the author's book on "Higher technical mechanics," where extensive use is made of these principles.

M. Hetenyi, USA

**Book—4843. Szabó, I., Higher technical mechanics [Höhere Technische Mechanik]**, 2nd ed., Berlin, Springer-Verlag, 1958, xii + 498 pp. DM 31.50.

This is the textbook for the second half of a four-semester-course sequence in mechanics, given by the author to all students of civil, mechanical and electrical engineering, and to those majoring in mathematics (!), at the Technical University of Berlin. It is probably the most comprehensive single volume on mechanics in its broadest interpretation, and even the tentative incorporation of this material into standard U. S. undergraduate curricula will appear rather utopian to most educators here.

The first of the four main chapters of the book is devoted to a thorough discussion of the basic principles of mechanics (132 pages). From the principles of virtual work and of D'Alembert, the Hamiltonian principle and the Lagrangian equations are derived, leading to a unified treatment of statics and dynamics, and resulting in a variety of methods of solution in the mechanics of rigid and deformable bodies (Castigliano's theorems, Rayleigh's Quotient, Ritz's method, etc.). The second chapter (187 pages) deals with selected topics from the theory of elasticity, and thin plates and shells, including respective problems in vibration, impact and elastic stability. The third chapter (70 pages) comprises a review of the theory of plasticity, with applications to plastic bending and torsion, inelastic buckling, and the rolling process. The last chapter (183 pages) deals with the theory of liquids and gases. Here a traditional treatment of potential flow theory is followed by a discussion of viscous fluids and the Navier-Stokes equations, while the second half of the chapter is devoted to problems of gas dynamics, including the use of potential theory and discussing supersonic flow and shock waves.

The advanced character of the book is most strikingly revealed in its illustrative problems, 73 of which are distributed among the chapters, while 11 more are included in the appendix. These problems are by no means of the type one can ordinarily expect to find in a textbook, and their solutions are fully discussed in each case. For instance, the treatment of a nonlinear buckling problem is illustrated by alternate methods of iteration and perturbation; among the plate problems a solution is derived for a dynamic deflection (and eventual instability) of a plate under a moving load; and one of the examples deals with thermal stresses in case of a material with temperature-dependent properties.

The book may also serve as a reference work or a refresher for those in the profession. As a result of the traditionally excellent Springer style of printing, and due to the fact that some of the text is in small type, its 500 pages comprise much more material than what would otherwise be expected from a volume of this size. It is a remarkably inclusive and coherent treatment of applied mechanics on the whole, and can be strongly recommended to any student, engineer or practitioner of the technical sciences.

M. Hetenyi, USA

**4844. Bordovskii, P. V., Problem of the rectilinear motion of a point of a variable mass in a resisting medium** (in Russian), Nauch. Tr. Odessk. Vyssh. Morekhodn. Uch-shche no. 2, 167-175, 1956; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 2571.

For the rectilinear motion of a point of a variable mass an integration of Meshcherskii's equation is carried out for the case of constant relative velocity of the flow of particles and the quadratic principle of resistance. In so doing it is assumed that the exponential principle of combustion is operating and no account is taken of the influence of the force of gravity. In a somewhat more general framework of assumptions this problem was solved by V. V. Beletskii [*Prikl. Mat. Mekh.* **20**, 4, 559-560, 1956].

V. S. Novoselov

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4845. Novoselov, V. S., Some problems on the mechanics of variable masses, taking into consideration the internal motion of the particles, II** (in Russian), *Vestnik Leningr. In-ta.* no. 1, 130-140, 210, 1957; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 2572.

For part I see *Vestn. Leningr. In-ta* no 19, 100-113, 1956.

The principle is established of the change of kinetic energy for a system and for body of variable mass. For such systems with holonome and nonholonome ideal linkages a general equation of mechanics is recorded. Lagrange equations of the second order are deduced for holonome systems with ideal linkages. "Additive forces" are determined; these must be added to the generalized active forces when these equations are recorded. A generalized Lagrange function is introduced and a generalization of the Hamilton-Ostrogradskii principle is recorded. Examples are examined showing the application of the deduced equations.

I. V. Livartovskii

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4846. Khailis, G. A., Some problems in statics and dynamics concerning the rotation of a wheel with slip operating** (in Russian), *Sb. Trudovpo Zemledel'cheskoi Mekhanika*, Moscow, Sel'khosgiz **3**, 400-476, 1956; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 2641.

The equation is recorded of the driven wheel taking into account the slip through and the resistance of the soil. To meet the case of even motion of the wheel an approximate determination is made of the dependence of the coefficient of slip on its influencing factors. There are misprints.

E. N. Berezkin

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4847. Mal'tsev, V. F., The theory of the process of the jamming of a roller mechanism of free motion** (in Russian), *Trudi Odessk. Tekhnol. In-ta* **8**, 69-73, 1957; *Ref. Zh. Mekh.* no. 4, 1958, Rev. 3746.

The theory is explained of the process of self-jamming of a roller mechanism of free motion. By analysis of the combination of forces acting on roller ($P$) during the process of self-jamming, the conditions were established for the self-jamming of slow-mov-

pressure, centers of gravity, pressure and buoyancy, etc.). The energy principles in statics are treated in detail (work, principle of virtual work, kinds of equilibrium positions, potential energy and the theorems regarding their extremal property). The next two chapters take up the kinematics and dynamics of a particle (kinematic quantities and their vector interpretations, straightline and curvilinear motion, Newton's laws of motion, dimensions and units, gravity and inertial forces, planetary motion). Chapter 10 deals with momentum principles (theorem of momentum, the law of momentum, collision of two balls, rocket flight in empty space, the applications in fluid mechanics—force of a jet, turbine blades, etc.). In the next two chapters are treated kinematics and dynamics of rigid bodies, especially translation, plane motion and rotation about a fixed axis with the theory of moments of inertia. In the next chapter the concepts of work and energy are applied to dynamical problems, e.g. flow on machines. Chapter 14 is devoted to the fundamentals of the theory of vibrations (free and forced). Several elementary technical problems are solved (e.g. vibration of a gear and of valve mechanism). The last chapter presents the introductory treatment of kinematics and dynamics in three-dimensional space by means of vector analysis. General theories of relative motion and moments of inertia are given. The gyroscopic problem is treated also, but regretfully without elementary technical applications.

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D. Raskovic, Yugoslavia

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The following cases of motion were examined:

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A. I. Zenkin

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H. Kolsky, England

**Book—4841. Szabo, I., Introduction to technical mechanics [Einführung in die Technische Mechanik, dritte verbesserte und erweiterte Auflage].,** 3rd ed., Berlin, Springer-Verlag, 1958, xii + 419 pp. DM 22.50.

The material contained in this volume is taught to all engineering students in the first two semesters at the Technical University of Berlin. It is a thorough and comprehensive treatment of statics and dynamics in solid and fluid mechanics. To cover such broad subject in one year's time is a remarkable educational achievement, which is made feasible only by the use of such a superior text as this one.

The subject matter of the book is divided into four principal chapters. The first one (75 pages) deals with the statics and dynamics of rigid bodies. A concise and self-contained account is presented at the outset of the elements of vector calculus and, from then on, vectorial treatments are given preference in the discussions through the rest of the book. The second chapter (65 pages) contains an introduction to stress analysis. Here the author circumvents entirely the traditional restrictive approach made in elementary strength of materials (which would have to be modified or discarded later on), and starts with the basic equations of the theory of elasticity. Nothing is sacrificed, however, by this generality of approach, and in this relatively short chapter the discussion goes so far as to include such topics as beams on elastic foundation, center of shear, and a variety of torsional problems. After the short third chapter (18 pages) dealing with the statics of composite systems, the last chapter (that is practically the second half of the book) is devoted to dynamics. The wide scope of this chapter may be best illustrated by the titles of its eight subdivisions, which are: kinematics, Newton's law and its consequences, the kinematics of the gravity center, sources of resistance to motion (friction), simple vibration problems, impact,

an outline of hydromechanics, and the fundamental laws of similitude.

Among the many virtues of this book two may be mentioned in particular, since they are of unusual pedagogical value. The first one is the author's inclusion of historical references in technical discussions, chiefly because he found that such comments are apt to arouse the students' interest and participation. The second feature is the lucid but intentionally terse style of presentation, thereby expecting and prompting the student to do his own share of thinking, which is by far the greatest educational service. The best evidence for the wide and well-deserved acceptance of this book is the fact that it reached three editions in less than five years. This last edition differs from the preceding one only by the addition of an appendix containing 15 representative problems with their solutions.

M. Hetenyi, USA

**Book—4842. Szabo, I., Introduction to technical mechanics, [Einführung in die Technische Mechanik, vierte erweiterte Auflage]**, 4th ed., Berlin, Springer-Verlag, 1959, xii + 434 pp. DM 22.50.

This latest edition is enhanced by a complementary chapter on the work-equations of mechanics (principles of virtual work and of D'Alembert). The purpose of this addition was to provide an introduction to this subject for those students who might not continue their studies with the author's book on "Higher technical mechanics," where extensive use is made of these principles.

M. Hetenyi, USA

**Book—4843. Szabó, I., Higher technical mechanics [Höhere Technische Mechanik]**, 2nd ed., Berlin, Springer-Verlag, 1958, xii + 498 pp. DM 31.50.

This is the textbook for the second half of a four-semester-course sequence in mechanics, given by the author to all students of civil, mechanical and electrical engineering, and to those majoring in mathematics (!), at the Technical University of Berlin. It is probably the most comprehensive single volume on mechanics in its broadest interpretation, and even the tentative incorporation of this material into standard U. S. undergraduate curricula will appear rather utopian to most educators here.

The first of the four main chapters of the book is devoted to a thorough discussion of the basic principles of mechanics (132 pages). From the principles of virtual work and of D'Alembert, the Hamiltonian principle and the Lagrangian equations are derived, leading to a unified treatment of statics and dynamics, and resulting in a variety of methods of solution in the mechanics of rigid and deformable bodies (Castigliano's theorems, Rayleigh's Quotient, Ritz's method, etc.). The second chapter (187 pages) deals with selected topics from the theory of elasticity, and thin plates and shells, including respective problems in vibration, impact and elastic stability. The third chapter (70 pages) comprises a review of the theory of plasticity, with applications to plastic bending and torsion, inelastic buckling, and the rolling process. The last chapter (183 pages) deals with the theory of liquids and gases. Here a traditional treatment of potential flow theory is followed by a discussion of viscous fluids and the Navier-Stokes equations, while the second half of the chapter is devoted to problems of gas dynamics, including the use of potential theory and discussing supersonic flow and shock waves.

The advanced character of the book is most strikingly revealed in its illustrative problems, 73 of which are distributed among the chapters, while 11 more are included in the appendix. These problems are by no means of the type one can ordinarily expect to find in a textbook, and their solutions are fully discussed in each case. For instance, the treatment of a nonlinear buckling problem is illustrated by alternate methods of iteration and perturbation; among the plate problems a solution is derived for a dynamic deflection (and eventual instability) of a plate under a moving load; and one of the examples deals with thermal stresses in case of a material with temperature-dependent properties.

The book may also serve as a reference work or a refresher for those in the profession. As a result of the traditionally excellent Springer style of printing, and due to the fact that some of the text is in small type, its 500 pages comprise much more material than what would otherwise be expected from a volume of this size. It is a remarkably inclusive and coherent treatment of applied mechanics on the whole, and can be strongly recommended to any student, engineer or practitioner of the technical sciences.

M. Hetenyi, USA

**4844. Bordovskii, P. V., Problem of the rectilinear motion of a point of a variable mass in a resisting medium** (in Russian), Nauch. Tr. Odessk. Vyssh. Morekhodn. Uch-shche no. 2, 167–175, 1956; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 2571.

For the rectilinear motion of a point of a variable mass an integration of Meshcherskii's equation is carried out for the case of constant relative velocity of the flow of particles and the quadratic principle of resistance. In so doing it is assumed that the exponential principle of combustion is operating and no account is taken of the influence of the force of gravity. In a somewhat more general framework of assumptions this problem was solved by V. V. Beletskii [*Prikl. Mat. Mekh.* **20**, 4, 559–560, 1956].

V. S. Novoselov

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4845. Novoselov, V. S., Some problems on the mechanics of variable masses, taking into consideration the internal motion of the particles, II** (in Russian), *Vestnik Leningr. In-ta.* no. 1, 130–140, 210, 1957; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 2572.

For part I see *Vestn. Leningr. In-ta* no 19, 100–113, 1956.

The principle is established of the change of kinetic energy for a system and for body of variable mass. For such systems with holonome and nonholonome ideal linkages a general equation of mechanics is recorded. Lagrange equations of the second order are deduced for holonome systems with ideal linkages. "Additive forces" are determined; these must be added to the generalized active forces when these equations are recorded. A generalized Lagrange function is introduced and a generalization of the Hamilton-Ostrogradskii principle is recorded. Examples are examined showing the application of the deduced equations.

I. V. Livartovskii

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4846. Khailis, G. A., Some problems in statics and dynamics concerning the rotation of a wheel with slip operating** (in Russian), *Sb. Trudovpo Zemledel'cheskoi Mekhanika*, Moscow, Sel'khosgiz **3**, 400–476, 1956; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 2641.

The equation is recorded of the driven wheel taking into account the slip through and the resistance of the soil. To meet the case of even motion of the wheel an approximate determination is made of the dependence of the coefficient of slip on its influencing factors. There are misprints.

E. N. Berezkin

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4847. Mal'tsev, V. F., The theory of the process of the jamming of a roller mechanism of free motion** (in Russian), *Trudi Odessk. Tekhnol. In-ta* **8**, 69–73, 1957; *Ref. Zh. Mekh.* no. 4, 1958, Rev. 3746.

The theory is explained of the process of self-jamming of a roller mechanism of free motion. By analysis of the combination of forces acting on roller ($P$) during the process of self-jamming, the conditions were established for the self-jamming of slow-mov-

ing roller mechanisms of free motion

$$\alpha \leqslant 2 \operatorname{arctg} \{(2rf_1f_2 + K_2f_1 - K_1f_2)/[r(f_1 + f_2) + K_2 - K_1]\}$$

where $\alpha$ is the angle of jamming, $r$ the radius of $P$, $f_1$ and $K_1$ the coefficients of slip friction and of free play friction on the surface of contact of $P$ with the external bearing, $f_2$ and $K_2$ the coefficients of slip friction and of free play friction on the surface of contact of $P$ with the internal bearing. Tests carried out by the author showed the tendency of roller mechanism to self-jamming, and compared satisfactorily with the proposed formulas. Some critical remarks are furnished regarding the existing theory of self-jamming.

L. K. Gordienko

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4848. Romiti, A., General method for the analysis and computation of block-type drum brake** (in Italian), *Atti Accad. Sci., Torino* **91**, 412, 24 pp., 1957.

Author has advanced an interesting and useful method of block-type brake analysis, by showing how a complicated braking system can be represented by an equivalent structural frame with regard to the forces acting on the system. The equivalent structure transforms difficult-to-solve braking problems into simple expressions and provides immediate solutions of most common braking problems. The equivalent structure also permits a rapid examination of the factors which determine the operation of the blocks in a drum-brake system.

The hypotheses used in this analysis are those of Reye (1860) on the proportionality of pressure and wear and the hypothesis that the blocks move as rigid bodies and the displacement of their points are congruent withthe condition that contact is maintained throughout the length of its lining. For statically indeterminate frames the problem is solved by assuming that the displacements are compatible with the geometry of the system.

Author explains the method of setting up the equivalent frame and the treatment of indeterminate systems. Many examples illustrate and support the analytical derivations.

M. Martellotti, USA

**Book—4849. Wolf, A., Fundamentals of planetary gears [Die Grundgesetze der Umlaufgetriebe]** 2nd enlarged ed., Braunschwieg, Germany, Friedr. Vieweg & Sohn, 1958, vii + 96 pp.

Book treats new computation methods for epicyclic gear trains. These methods are not connected to any form of the gear train or case of load, but are applied generally to give design data. Former computations started from the steady gear trains and used graphical procedures for the determinations of revolutions and torques. With the new symbolic calculation shapes the main data are given clearly and new simple laws are developed. The epicyclic gear trains are considered as torque converters between three shafts, where the steady gear train is only a special case. Book contains chapters on: the special and general computations of steady gear trains, the gear-ratios of epicyclic gear trains for two shafts (internal) and three shafts (external), the influence of load but without losses, loaded epicyclic gear trains with losses considered, simply paired and multifold coupled epicyclic gear trains. The laws are compiled at the end, and 18 numerical examples give a clearer view. All quantities included in the laws are dimensionless. Book is a valuable aid for designers, but due to the many symbols is not so easily understood.

N. Theophanopoulos, Greece

**4850. Katz, E., and de Pater, A. D., Stability of lateral oscillations of a railway vehicle,** *Appl. Sci. Res.* (*A*) **7**, 6, 393–407, 1958.

The lateral motions of a highly simplified model of a railway vehicle on a straight track are studied. The asymptotic form of the periodic motion for high and for low (near-critical) speeds is developed. The stability of the periodic motion is investigated. It is found that any motion starting with small deviations from the periodic motion approaches the latter in the course of time. This approach is slow for very high speeds and for near-critical speeds, but fast for intermediate speeds. Theoretical results for highly simplified model conform closely to reality.

From authors' summary by E. G. Newman, USA

**4851. Hsu, C. S., On simple subharmonics,** *Quart. Appl. Math.* **17**, 1, 102–105 (Notes), Apr. 1959.

Rosenberg has shown that periodic solutions of the class of differential equations of the type $\frac{d^2x}{dt^2} + \frac{x}{r^2} + k\sum_m^r a_r^n x^n = k \cos t$, ($r$ 1, 2, 3, ...) are of the simple subharmonic type $x = \cos(t/r)$. Using a transformation with Tchebyshev polynomials, present author shows that these differential equations and their subharmonic solutions may be derived from the linear equation $\frac{d^2x}{dt^2} + (1 + k)\,x = k \cos t$ and the corresponding periodic solution $x = \cos t$. Uniqueness of the simple subharmonic steady-state solution is established, subject to continuity of the solution.

A. P. Boresi, USA

**4852. Gotusso, G., Stability of mechanical systems in movement** (in Italian), *Atti Accad. Naz. Lincei, R. C. Cl. Sci. Fis. Mat. Nat.* (8) **24**, 6, 693–700, June 1958.

Author considers the motion $M$ of a holonomic or nonholonomic mechanical system such that $\omega_1 = K$, $\omega_2 = 0$ ..... $\omega_n = 0$ ($\omega_1$ $\omega_2$ .... $\omega_n$ are $n$ linear independent combinations of $q_1$, $q_2$ .. $q_n$, $q_1$, $q_2$ $q_n$ are the coordinates of the system, $K$ is a very large number) and he proves that to bring the system in a motion adjacent to $M$ it is necessary, in general, to increase the forces of a quantity proportional to $K$. The stability of a gyroscope is an example of this theorem.

D. Graffi, Italy

## Instrumentation and Automatic Control

(See also Revs. 4829, 5135)

**Book—4853. Grabbe, E. M., Ramo, S., and Wooldridge, D. E., Handbook of automation, computation, and control—Control fundamentals, Vol. I,** New York, John Wiley and Sons, Inc., 1958, xx + 946. $17.

"Control fundamentals" was prepared to enable engineers, scientists, and computer and control technicians to keep ahead of the rapid developments in the automation and control field. As such it is directed to the problem solvers and technical management who are concerned with applying technology to the practice of automatic controls. So rapidly have control devices been evolved that in order to keep up with an understanding of the technical know-how and to form the basis for further progress it has become necessary to compile and condense basic concepts, mathematical tools and accepted techniques.

"Control fundamentals" consists of a collected set of chapters of text by twenty nine recognized authorities in control theory, which were edited and welded together into a comprehensive handbook by Drs. Grabbe, Ramo, and Wooldridge. The contributors are all practicing experts working in leading universities and in industry. The material is arranged in such a way that one may review any chapter without recourse to previous chapters, as in a true handbook. Each chapter starts with definitions and descriptions

aimed at providing background and then develops more complicated theory and applications.

The handbook begins with fundamental mathematical concepts, such as Sets, Algebraic equations, Matrix theory, Differential and integral equations, Probability, Complex variables, LaPlace transforms, and many others. This material is followed by Numerical analysis, Operations research, Information theory and transmission, and Feedback control. Even though the presentation is quoted by the editors as presuming some engineering training, the level of mathematics is essentially senior college or first-year graduate school. It should be mentioned, however, that the emphasis is on presentation of working mathematical forms rather than on comprehensive derivations and rigor. The chapters on general mathematics are strong in the treatment and include chapters on subjects not ordinarily found in engineering handbooks. More than half the book is devoted to the fields of information theory and transmission, and feedback control, and the work includes a summary of the present status of feedback-control theory covered in eight chapters. The material is presented in a clear and comprehensible way which endows the book with a rich background of fundamental information in fields which are becoming more significant.

The material on numerical analysis is quite up to date and represents real insight into modern requirements for high-speed digital computing. The section on operations research is essentially a distillation of earlier work by scientists at Case Institute. Admittedly, this area of control is difficult to describe, because its effect on management has been a subject of controversy for some time.

The Handbook has succeeded in presenting the essential theory behind today's rapid developments in the automation and the control field. By so doing, it takes an important place in providing basic information for new fields. J. W. Fischbach, USA

**Book—4854. Truxal, J. G., edited by, Control engineers' handbook—Servomechanisms, regulators, and automatic feedback control systems,** New York, McGraw-Hill Book Co., 1958, xiii + 1030 pp. + index. $18.50.

Handbook was reviewed by reading sections and parts of sections in an arbitrary sequence in order to simulate ordinary usage. In every case reviewer found well-prepared and useful information. Handbook contains theoretical and mathematical treatment of control system analysis and synthesis techniques as well as physical and practical explanations of components. Information ordinarily contained in a rather extensive library is available here in a centralized form, presented in a concise manner.

Handbook was prepared by a group of 36 specialists. The following list of section titles indicates the breadth of coverage: Tables and mathematics, Basic feedback-control-system theory, Design techniques, Specialized design techniques, Computers in control, Electronic control elements, Magnetic amplifiers, Transistors and miniaturization, Thyratron amplifiers, Contactors and relays, Electric power supplies, Electromechanical actuators, Mechanical components, Clutches and brakes, Hydraulic controls, Pneumatic components, Signal transducers, Regulators, Process-control systems. Handbook is conveniently indexed.

The part on random functions and statistical methods is reviewed as an example of the theoretical treatment. This part did not contain sufficient material to be used as a text by the reader not acquainted with the subject; the uninitiated reader would be able to obtain a reading knowledge of statistical theory but not a working knowledge. On the other hand, a reader with previous background wishing to review or reference some point would find this section exceedingly useful.

As an example of a component discussion the keyed demodulator (Schreiner circuit) is cited. The explanation of the operation of this component was inadequate; however, a suitable source for this information was referenced. Of more importance, a circuit diagram with element values of a demodulator-modulator with a d-c stabilization network is given. A study of the circuit usually yields knowledge of operation; however, the inverse process is usually more difficult.

Reviewer feels that every control engineer and every engineer and scientist who has work dealing with control problems will wish to have this handbook available for reference purposes.

V. B. Haas, Jr., USA

**Book—4855. Rothstein, J., Communication, organization, and science,** Indian Hills, Colo., Falcon's Wing Press, 1958, xcvi + 110 pp. $3.50.

This book is concerned with the new and large complex of which automation and cybernetics are parts. Its chief aim is to show how the concepts of entropy and organization are related to measurement and communication. Author presents a broad and, in general, elementary introduction to this vast field; clearly written, the book seldom uses highly technical developments and is thus accessible to a reader with a general knowledge of thermodynamics and elementary mathematics.

A list of the chapters is the best way to give the scope of the content: 1. Information and entropy; 2. Measurement and communication; 3. Information and thermodynamics; 4. Information, logic, and physics; 5. Language, science, and art; 6. Information, organization, and systems; 7. Theory as organization of observation; 8. Skill as organization; 9. Miscellaneous applications of the organization concept; 10. Science, values, and games; 11. On the behavior of organized mechanisms; 12. On the possibility of constructing a universal language; and 13. Toward a unified world outlook.

Even in the purely expository parts the author (who seems to be well informed, except possibly on the developments within the last 5 years) often added interesting personal insights; to the reviewer his most interesting contribution is in Chapter 11. The book has a lengthy "Foreword", by C. A. Muses; the reviewer refrains from any opinion on this introduction, feeling incompetent on what he considers as metaphysics (e.g., on p. lvi the complex numbers $a + bi$ are discussed with reference to the survival of the psyche after destruction of the body). J. Kampe de Feriet, France

**4856. Chirlian, P., Bounds on the error in the unit step response of a network,** *Quart. Appl. Math.* **16,** 4, 432–435 (Notes), Jan. 1959.

Author presents equations which reduce the problem of correlating the transfer function and transient response of a system particularly when it is desirable to simplify the transfer function for purposes of calculation. When calculating a unit step response from a transfer function, one would be able to determine the maximum error in the calculated step response for assuming that the transfer function is zero beyond some arbitrary frequency. With this knowledge, considerable simplification of the calculations can often be effected. Equations are also given which determine the amplitude and phase error effects upon the calculated unit step response when using a simplified transfer function for calculations.

Use of these equations can simplify the design procedure without materially changing the transient response characteristics of a network or system. W. R. Deazley, USA

**4857. Kessler, C., The symmetric optimum, Parts I and II** (in German), *Regelungstech.* **6,** 11, 395–400, Nov. 1958; **6,** 12, 432–436, Dec. 1958.

Conjunction of consideration of the relative magnitudes of the time constants of the open-loop transfer function and rather simple analysis yields equations and associated rules enabling single-parametric adjustments of simple closed-loop controllers used in process control work, such that "optimum" (i.e., reasonable overshoot, rapid settling) of the transient response results. Verification of the corrections of analytic derivations and illustrations of

the utility of the procedures advanced are supported by numerous Siemen's analog-computer-obtained oscillographic records. This paper merits reading by all directly concerned with process control instrumentation and theory. T. J. Higgins, USA

**4858. Balchen, J. G., A performance index for feedback control systems based on the Fourier transform of the control deviation,** *Acta Polytech. Scandinavica* no. 247, MA 1, 18 pp., 1958.

After a review of commonly used performance indices for optimum adjustment of linear control systems based on integrals of functions of the deviation in response to a specified disturbance, author suggests a new index. This index is the maximum value of the modulus of the Fourier transform of the deviation; it can be derived from a graphical construction based on a modification of the Nichols diagram. Author shows how the construction can be modified to ensure adequate damping either by the introduction of a constraint on the loop transfer function or by the use of an index which involves also the rate of change of deviation.

G. D. S. MacLellan, England

**4859. Kurakin, K. I., On the selection of optimum characteristics of linear servomechanisms, Part II** (in Russian), *Avtomatika i Telemekhanika* **17,** 7, 648–663, July 1956.

The optimum transfer function is determined for a servo system trying to follow a random signal with spectral density represented by $\frac{N}{\omega^4(a^2+\omega^2)}$ in the presence of white noise. The characteristics of the optimum system are calculated by conventional methods. Most of the paper treats the problem of approximating the optimum system by linear physically realizable elements. Passive R-C and R-L-C electronic networks are described for achieving this result, and also an equivalent mechanical system. Values of the components are then computed which approximate the optimum system and are readily obtainable.

Some of the procedures are of general usefulness, such as the methods of analyzing the performance of the resulting system. The synthesis of components to give the desired performance is treated largely as an art of selecting those which give good performance, are simply arranged, and easily obtained. The suggestions in this regard which are put forth in the process of working out specific examples probably were more original in 1955 when the paper was written than they would be considered today.

L. H. Schindel, USA

**4860. Jury, E. I., and Semelka, F. W., Time domain synthesis of sampled-data control systems,** *Trans. ASME* **80,** 8, 1827–1838, Nov. 1958.

Paper deals with the synthesis of compensating (equalizing) networks for sampling servos which impart certain specified features to the transient responses from representative inputs. The latter are the unit step, unit ramp, unit parabola, etc. The response features are typically overshoot, peak time, settling time, and steady-state error. Discrete and continuous compensating networks are considered. A procedure is evolved which, when supplemented with some engineering judgment and physical insight, provides the desired network parameters. Two concrete and detailed sample applications of the procedure are exhibited.

R. F. Drenick, USA

**4861. Hofmann, R., Contribution to the calculation of the quadratic control area** (in German), *Regelungstech.* **6,** 4, 142–145, 1958.

Paper shows that it is possible to obtain a closed expression for the quadratic control areas without knowing the zeros of $N(s)$. This expression is obtained by using the coefficients $a_j$, $b_j$ of the Laplace transform $f(s)$. The calculations are not too difficult except that in case of large $n$ they are very time consuming. Examples are given. P. P. Biringer, Canada

**4862. Gibson, J. E., Nonlinear systems design, Part II: How to interpret the phase-plane plot,** *Control Engng.* **6,** 1, 85–88, Jan. 1959.

**4863. Gibson, J. E., Nonlinear systems design, Part III: An introduction to synthesis on the phase plane,** *Control Engng.* **6,** 2, 82–85, Feb. 1959.

**4864. Tomizuka, C. T., and Zimmerman, D., Improvements on a simple temperature controller,** *Rev. Sci. Instrum.* **30,** 1, p. 40 (Notes), Jan. 1959.

**4865. Valstar, J. E., A fresh look at selecting control valve characteristics,** *Control Engng.* **6,** 3, 103–108, Mar. 1959.

**4866. Skoog, J. A., and Setterlund, G. G., Space requirements for equipment items subjected to random excitation,** *J. Acoust. Soc. Amer.* **31,** 2, 227–232, Feb. 1959.

A gentle excursion into the theory of stationary random time series as applied to estimating the amplitude of vibrations in linear mechanical supports. The role and limitations of statistical concepts is extremely obscure, and the calculations illustrated are a trivial extension of standard Fourier series analysis.

R. M. Stewart, USA

**4867. Dorrance, W. H., Hypersonic aerodynamic factors in performance, guidance, and control,** *Aero/Space Engng.* **17,** 5, 30–33, 43, May 1958.

**4868. Fletcher, J. C., Guidance and control problems in the Air Force ballistic missile program, Part I,** *Aero/Space Engng.* **17,** 5, 50–54, May 1958.

**4869. Wilson, R. E., and Lewis, J. B., Cross coupling in inertial navigation systems,** *Aero/Space Engng.* **17,** 5, 60–65, 80, May 1958.

**4870. Powell, R. W., and Kauffman, W. M., Infrared application to guidance and control,** *Aero/Space Engng.* **17,** 5, 66–71, May 1958.

**4871. Feng, T. Y., Static and dynamic control characteristics of flapper-nozzle valves,** ASME Ann. Meet., New York, N.Y., Nov./Dec. 1958. Pap. 58-A-160, 7 pp.

**4872. Webster, W. M., Continuous-path numerical control,** ASME Ann. Meet., New York, N.Y., Nov./Dec. 1958. Pap. 58-A-162, 4 pp.

**4873. Childs, J. J., Operating experience with numerical control,** ASME Ann. Meet., New York, N.Y., Nov./Dec. 1958. Pap. 58-A-238, 5 pp.

**4874. Crabtree, J., Automatic programming methods for numerical control,** ASME Ann. Meet., New York, N.Y., Nov./Dec. 1958. Pap. 58-A-268, 7 pp.

**4875. Sedgwick, R. K., Automation approach using numerical control,** ASME Ann. Meet., New York, N.Y., Nov./Dec. 1958. Pap. 58-A-288, 4 pp.

**4876. Tilton, P. D., Retrofit applications of numerical controls to existing machines,** ASME Ann. Meet., New York, N.Y., Nov./Dec. 1958. Pap. 58-A-290, 12 pp.

**4877. Stocker, W. M., Jr., Experience with discrete-positioning numerical control in the aircraft industry,** ASME Ann. Meet., New York, N.Y., Nov./Dec. 1958. Pap. 58-A-293, 13 pp.

**4878. Hoover, G. W., Man-machine integration in space vehicles,** *Aero/Space Engng.* **18,** 2, 54–58, Feb. 1959.

**4879. Christoph, P., An 84-minute tuning in gyro- and space-compasses** (in German), *Ing.-Arch.* **26,** 4, 233–241, Aug. 1958.

Based upon exact vector analysis the necessary condition is worked out for gyro- and space-compasses to be free from the effects of vehicle acceleration. The condition reduces into 84-minute tuning found by M. Schuler in 1923; several methods are proposed and discussed concerning its realisation. The space-compass is defined here as a compass giving not only north but horizon by satisfying another condition. The space-compass may serve as a measuring system for inertial navigation.

M. Sanuki, Japan

## Elasticity

(See also Revs. 4933, 4937, 4939, 4942, 4971, 5023, 5033, 5046, 5050, 5057)

**Book—4880. Dinnik, A. N., Papers on the mining industry** (in Russian), Moscow, Ugletekhizdat, 1957, 195 pp. + illus. 6r. 90k; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 3303.

Papers included in the compendium deal with various matters concerning the mining industry: the application of the theory of elasticity to the solution of problems related to management of mine roofs; the dynamic stresses in hauling cables, the strength of hauling cables; the danger of resonance in winding machinery equipped with bicylindro-conical drums, the distortion of mine shafts, etc. The compendium does not include papers bearing on the subject published by A. N. Dinnik, as for instance: On the vibration of a wire of varying toughness [*Izv. Ekaterinoslavsk. Gorn. In-ta*, 1914, no. 1]; Dynamic stresses in a hauling cable when the top end unexpectedly stops [*Yuzh. Inzhener.* 1917, nos. 3, 4]; Dynamic stresses in a hauling cable during an evenly accelerated motion of its top end [*Byul. Ekatermoslavsk. Gorn. In-ta*, 1918]; Dynamic stresses in a hauling cable [Nauka na Ukraine, 1922].

G. N. Savin

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4881. Bernstein, B., and Ericksen, J. L., Work functions in hypo-elasticity** (in English), *Arch. Rational Mech. Analysis* **1,** 5, 396–409, Aug. 1958.

The work done by the stresses in deforming a perfectly elastic body depends only on the initial and final state of the body. In the theory of hypoelasticity proposed by C. Truesdell [*J. Rational Mech. Anal.* **4,** 83–133, 1955; AMR **9** (1956), Rev. 2853] however, the work may be a functional of the entire history of the stress. Authors study two special types of hypoelastic materials in which the work functional reduces to a function of the initial and final stress only.

Materials of type I are characterized by the existence of a distinguished initial stress such that the work done is nonnegative for every deformation starting from a state with the distinguished stress. For such materials, the authors show that the distinguished stress must be zero and that the work per unit final volume depends only on the final stress provided the deformation starts from an unstressed state.

Materials of type II are characterized by the property that the work done is nonnegative for all deformations in which the initial and final stresses coincide. For such materials, the work per unit initial volume depends only on the initial and final stresses.

W. Noll, USA

**4882. Knops, R. J., On the variation of Poisson's ratio in the solution of elastic problems,** *Quart. J. Mech. Appl. Math.* **11,** 3, 326–350, Aug. 1958.

Particular values of Poisson's ratio (e.g. 0, ½, ±1, ∞) can effect a significant reduction to the field equations (e.g. Potential theory for $\nu = \infty$). If in such cases the solution of the boundary-value problem is known, field equations with the boundary conditions zero are established to complete the solution for a general $\nu$. Method employs Maxwell stress functions including Westergaard's twinned gradient method. Discussed problems include: internal force in semi-infinite body with rigid boundary [Rongved, AMR **9** (1956), Rev. 3197], hollow cone loaded at the vertex [Neuber, *ZAMM* p. 203, 1934].

H. Schaefer, Germany

**4883. Korifel'd, M., Methods and results of an investigation of the three-dimensional elasticity of materials** (in Russian), *Uspekhi Fiz. Nauk* **54,** 2, 315–342, 1954; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 3547.

Investigational methods used in ascertaining the three-dimensional elasticity of gaseous, liquid and solid elements at high pressures (up to 100,000 kg/cm²) are described. Details are also given of the designs of apparatus for the study of three-dimensional and linear elasticity of materials during all-sided pressure (piezometers-weighing, capillary and linear, with electric micrometers, lever devices, piston and bellows, and also different types of plunger piezometers). Results are furnished for investigations on the three-dimensional elasticity of materials carried out by several authors (principally by Bridgeman). It is shown that nearly all materials, over a wide range of pressures, possess a linear relation of the modulus of volumetric elasticity $K$ to pressure $p$, in agreement with which $K = x(p_* + p)$, where $x$ and $p_*$ are constants. The temperature relations of the modulus of volumetric elasticity are discussed.

Increasing the pressure of a series of materials discloses phase transformations. In phase transitions of the first type, which take place in consequence of crystallization or allotropic transformations, there is a formation of a jump-like change of volume. Phase transformations of the second type are accompanied by a jump-like change of the modulus of volumetric elasticity. Based on an analysis of the experimental data by Bridgeman, author makes an assumption regarding the possible existence of phase transformations of the third type, in which there is no volumetric jump or jump of the modulus of volumetric elasticity, whereas the characteristics $x$ and $p_*$ change in jump-like fashion. The volumetric and linear elasticity of anisotropic bodies subjected to all-sided pressure comes under discussion. A description is given of the anomalous volumetric elasticity of quartz glass and cerium.

N. I. Malinin

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4884. Goded, F., Spherical symmetry in theory of elasticity,** *J. Appl. Mech.* **25,** 1, 136–140, Mar. 1958.

See AMR **11** (1958), Rev. 1525.

**4885. Rongved, L., and Frasier, J. T., Displacement discontinuity in elastic half-space,** *J. Appl. Mech.* **25,** 1, 125–128, Mar. 1958.

See AMR **11** (1958), Rev. 3007.

**4886. Csonka, P., Additions to the elasticity of isotropic bodies** (in Hungarian), *Magyar Tud. Akad. Musz. Tud. Oszt. Közl.* **22,** 1–3, 75–76, 1958.

**4887. Gudushauri, I. I., Evaluation of reactive tangential stresses when making calculations for a strip of finite stiffness on an elastic semiplane** (in Russian), *Soobshch. Akad. Nauk*

*GruzSSR* **18**, 1, 67–74, 1957; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 3323.

The problem is examined regarding the deflection of a strip lying on an elastic semiplane, when there is adhesion along the whole line of contact. The normal and tangential stresses on the line of contact are sought in the form of

$$p(x) = \frac{1}{\sqrt{1-x^2}} \sum_{n=0}^{\infty} A_n T_n(x) \qquad [1]$$

$$t(x) = \frac{\xi}{\sqrt{1-x^2}} \sum_{m=0}^{\infty} B_m T_m(x) \qquad [2]$$

where $x$ is a nondimensional coordinate, $T_k$ is Chebyshev polynomial. Retaining in [1] and [2] the end number of terms, author obtains the essential number of equations for the determination of the unknowns $A_n$ and $B_m$ from the conditions of equality of the bending and elongation of the beam and boundary of the semiplane on the line of contact. The case is calculated when the strip is evenly loaded by a distributed load. It is explained that the adhesion between the sole of the strip and the surface of the base exercises considerable influence on the deflection moments and bending as regards their diminution. The influence of adhesion along the contact surface on the normal reactive stresses grows with increase in the flexibility of the strip, while for stiff strips it becomes practically imperceptible.

V. I. Mossakovskii

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4888. Vyaz'menskii, S., Basic equations of the nonlinear theory of small elastic deformations of thin three-dimensional rods** (in Russian), 15th Scientific Conference Leningrad Engng.-Constr. Inst., Leningrad, 1957, 367–371; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 3338.

A system is put forward, consisting of 12 nonlinear differential equations, characterizing the stressed and deformed state of a thin-walled rod of open profile, derived with consideration for the influence on the general stress-deformation state of the rod of constrained torsion. This system of equations, in the theory of small deformations, is in itself a substitute for the Klebsch-Kirchoff equations.

V. V. Vlasov

Courtesy Referativnyi Zhurnal, USSR
*Translation, courtesy Ministry of Supply, England*

**4889. Horvay, G., and Hanson, K. L., The sector problem,** *J. Appl. Mech.* **24**, 4, 574–581, Dec. 1957.

See AMR **11** (1958), Rev. 791.

**4890. Verma, G. R., Notes on the problem of a semi-infinite elastic solid with loads distributed along the circumference of a circle on the plane surface,** *Bull. Calcutta Math. Soc.* **49**, 4, 217–220, Dec. 1957.

The stress distributions of a semi-infinite elastic solid with axially symmetric loading have been expressed in an infinite integral involving Bessel's function of the first kind. In applying this method to the case with normal and shearing loads distributed along the circumference of a circle on the plane boundary of the solid, Dirac's delta function is used in the present note. The expressions for the stress distribution and displacements are readily obtained.

T. H. Lin, USA

**4891. Baldacci, R. F., On the variational principle of the theory of finite elastic deformation** (in Italian) *Atti Accad. Naz. Lincei R. C. Sci. Fis. Mat. Nat.* (8) **23**, 5, 250–254, Nov. 1957.

**4892. Rogozinski, M., An attempt to establish the theoretical foundations of the Moire method of strain and stress analysis** (in English), *Arch. Mech. Stos.* **9**, 2, 191–210, 1957.

**4893. Buglai, B. M., Surface deformation of wood under measured pressure** (in Russian), Derevoobrabat. Prom-st' no. 8, 5–9, 1956; *Ref. Zh. Mekh.* no. 11, 1957, Rev. 13600.

**4894. Dorn, W. S., and Schild, A., A converse to the virtual work theorem for deformable solids,** *Quart. Appl. Math.* **14**, 2, 209–213 (Notes), July 1956.

Simply connected region $V$ bounded by surface $S$ with outward unit normal $n_i$ is considered. Cartesian tensor notation is used. A vector $u_i$ defined on $S$ and a symmetric tensor $\epsilon_{ij}$ defined in $V$ are considered to be given. Following mathematical theorem is proved: If equation $\int \sigma_{ij}\,\epsilon_{ij}\,dV = \int u_i\,\sigma_{ij}\,n_j\,dS$ is satisfied for *all* symmetric tensors $\sigma_{ij}$ that are solutions of the differential equations $\sigma_{ij,j} = 0$, there exists a vector $u_i$ in $V$ that takes the given values on $S$, and that satisfies the differential equations $2\,\epsilon_{ij} = u_{i,j} + u_{j,i}$. This theorem may be used for testing compatibility of a given system of strains, although well-known differential equations of compatibility appear to be easier to apply. Authors suggest, however, that their theorem may be used in approximate way to determine "least incompatible" strain tensor in a given class of tensors.

Stress-strain relation is not involved in proof, since theorem relates explicitly to compatibility of strains. There is an allied theorem of complementary energy which requires hypothesis that strain-energy-density function exists, since it relates to compatibility of stresses. In referring to a proof of principle of complementary energy, authors remark, "Our theorem is not limited to linear elasticity theory." It is equally true that proof of principle of complementary energy to which they refer does not employ hypothesis of Hookean material. Applications of both theorems rest on linear geometric approximations of small-displacement theory, since equation $2\,\epsilon_{ij} = u_{i,j} + u_{j,i}$ does not determine strain tensor if strains or rotations are large.

H. L. Langhaar, USA

**4895. Brock, J. S., The stresses around square holes with rounded corners,** *J. Ship Res.* **2**, 2, 37–41, Oct. 1958.

Paper presents analytical solution for stress-concentration factor $K_t$ at boundary of square hole with rounded corners in infinite plate subjected to uniform tensile load. Data presented show variation of $K_t$ with ratio of notch radius $r$ to hole width $W$, for loading parallel to side and for loading parallel to diagonal of hole. Solution is intended for application to ship structures but extension to plates of finite width in order to cover perforated columns and beam webs, and ship plates with large openings, would seem desirable.

R. J. Roark, USA

**4896. Dutt, S. B., Stress concentrations around a small spherically isotropic spherical inclusion on the axis of an isotropic circular cylinder in torsion,** *J. Technol.* **3**, 1, 13–17, June 1958.

The radius of the inclusion is small with respect to the radius of the cylinder. Das has already dealt with the isotropic spherical (or spheroidal) inclusion [AMR **7** (1954), Rev. 1034]. Present paper is the specialization for the case where the inclusion is spherical isotropic, i.e. there is transverse isotropy about the radius vector [see Love in his chapter "Curvilinear aeolotropy"].

J. P. Benthem, Holland

**4897. Hicks, R., Asymmetrically ring reinforced circular hole in a uniformly end-loaded flat plate with reference to pressure vessel design,** Instn. Mech. Engrs., Prepr., 11 pp., 1958.

Author considers reinforcements of circular holes placed on one side of the plate, so that the other side remains flush. Such

a reinforcement introduces a bending state of the plate around the hole. Resulting maximum stress is then higher than for an equal but symmetrically placed reinforcement and author shows practical case where stress concentration is about as high as for an unreinforced hole, although reinforcement satisfies the British Standards Specification for Fusion-welded Pressure Vessels.

Paper includes data which enable designer to determine stress distributions associated with such reinforcements.

F. I. Niordson, Denmark

**4898. Szelagowski, F., The state of stress in a disc rotating about an eccentric axis** (in Polish), *Rozprawy Inz.* **6**, 3, 455–465, 1958.

The stress components (in polar co-ordinates) are determined in a disk rotating about an eccentric axis. The solution is obtained by superposing two states of stress. The first is that in an infinite rotating disk additionally loaded by a concentrated force (the results of one of the author's previous papers being used). The other (compensation) state of stress is that in a disk of radius $R$ with continuous loads on its periphery, differing only by the sign from those at the corresponding points of the infinite disk. The latter state of stress was determined using the familiar H. A. Schwartz equation. The equation obtained is accurate.

L. Martini, Poland

**4899. Choudhury, P., Two-dimensional thermal stress due to steady heat flow in a semi-infinite aeolotropic plate,** *J. Technol.* **3**, 1, 19–24, June 1958.

Thermal stress in a straight edge semi-infinite aelotropic plate was investigated by using Fourier integral representation for the stress function, integrated along the edge, and exponentially damped in the normal direction. This enables one to solve problems for arbitrary temperature distributions along a straight edge of the plate.

Misprints are noted in equations (3.2), 3.5), 3.6), where the dependent variable should be $\chi$.

Reviewer believes the use of "aelotropic" instead of "orthotropic" in the present case is an unfortunate loose nomenclature. Indeed, stress-strain relationships (1.1) and (3.1) hold along principal directions of orthotropic materials only.

H. S. Tan, USA

**4900. Johnson, C. H. J., The determination of the thermoelastic stress distribution in hollow cooled turbine blades,** *Aero. Res. Lab., Melbourne, Austral., Rep.* ME 87, 31 pp., June 1958.

This excellent report considers the problem of determining the steady-state thermal stresses in a two-dimensional hollow-turbine blade of arbitrary section with walls which are thin compared to the chord. The method for calculating the heat conduction in the blade and the thermal stresses is derived and an example is worked. To make the mathematics of the problem as simple as possible, the blade surface is divided into segments over which both the internal and external heat-transfer coefficients are assumed to be constant. Author derives a system of ordinary linear-differential equations which describes the heat conduction in the blade wall. Heat conduction in the trailing-edge region of the blade is also considered along with the boundary conditions. The turbine blade is considered as a semi-infinite rod with the blade section as the cross section of the rod. This rod is such that it is not subjected to any external forces and is clamped at one end. The total stress in the blade is considered to be the sum of an "elastic stress" and a "thermoelastic stress." In evaluating the constants used in the final stress equation, numerical integration is used for those which apply to the blade wall and Bessel functions are used to determine those which apply to the trailing-edge region.

G. R. Fusner, USA

**4901. Paria, G., Coupling of elastic and thermal deformations, Part I,** *Appl. Sci. Res. (A)* **7**, 6, 463–475, 1958.

Using the exact heat equation for the isotropic thermoelasticity, including the term due to the cubical dilatation, author determines the time-dependent distribution of stress, strain and temperature in a half space $z > 0$ due to an uniformly heated circular region of the plane $z = 0$. The case of a thermic nucleus is also examined. Hankel- and Laplace-transforms prove to be very useful, and the direct functions are approximately found by developing in series and taking the first terms.

T. Manacorda, Italy

**4902. Hammitt, F. G., Axial-temperature-gradient bending stresses in tubes,** *J. Appl. Mech.* **25**, 1, 109–114, Mar. 1958.

See AMR **11** (1958), Rev. 3479.

**4903. Stavsky, Y., Constrained torsion of an elliptical cylinder by forces applied on the lateral surface,** *Bull. Res. Counc. Israel* **6C**, 2, 127–134, Feb. 1958.

An approximate solution is given for an elastic cylinder of elliptic cross section which is twisted by uniform tangential tractions applied to its lateral surface, one terminal section being constrained against warping.

E. Sternberg, USA

**4904. Kurdin, N. S., On the approximate solution of differential equations in the theory of elasticity (the torsion problem)** (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 2, 133–135, Feb. 1958.

A finite differences method has been applied to nonlinear second-order differential equation $L[\Phi] = p$. Approximate values $\overline{\Phi}_{i,k}$ in the nodes $(i,k)$ of a rectangular network have been obtained by the use of an iterative procedure. The function $\Phi(x,y)$ was found by equating the second derivatives $\Phi''$ to the expressions of the type $(\overline{\Phi}_{i-1,k} - 2\,\overline{\Phi}_{i,k} + \overline{\Phi}_{i+1,k})/h^2$ instead of the known equality $\Phi(x_i, y_k) = \overline{\Phi}_{i,k}$. The accuracy of the method is estimated by the summation of the absolute values of the errors in the points of the network considered, instead of using the inequality $|L[\Phi] - p| < \varepsilon$.

A numerical example of a twisted bar with square cross section proves that the present method is more accurate than the one usually applied.

M. Sokolowski, Poland

**4905. Csonka, P., Numerical method for calculation of stresses due to torsion** (in Hungarian), *Magyar Tud. Akad. Musz. Tud. Oszt. Köz.* **21**, 1/4, 365–367, 1957.

**4906. Loo, T.-T., Effect of curvature on the Hertz theory for two circular cylinders in contact,** *J. Appl. Mech.* **25**, 1, 122–124, Mar. 1958.

See AMR **11** (1958), Rev. 2016.

## Viscoelasticity

**4907. Bychawski, Z., On the application of creep function in generalized form** (in English), *Bull. Acad. Polonaise Sci., Cl. (IV)* **5**, 2, 115–120, 1957.

Phenomena of creep and shrinkage deformation in the core of hooped concrete elements are expressed in terms of kernel function depending on time of observation and on time of loading. Using separable kernels of a type studied by E. Goursat [*Bull. Sci. Math.* **2**, p. 57, 1933] author obtains linear differential equation with constant coefficients for the corresponding resolving kernel. Special case of initial stress of straight cable prestressing prismatic concrete element is treated with kernel composed of two exponential terms and worked out in detail. Explicit formula for cable stress is given.

F. K. G. Odqvist, Sweden

**4908. Christy, R. W., Theory of creep limited by self-diffusion,** *J. Appl. Phys.* **30,** 5, 760–764, May 1959.

A theory of steady-state creep at high temperature and stress is based on the climb of dislocations limited by the diffusion of vacancies from them. The creep rate is found to be approximately $(MnL^3)D_0 \exp[-W + E + U \mp \Delta W)/kT]$, where $M$ is the number of dislocation sources per unit volume, $n$ is the number of dislocations of length $L$ per unit area, and $D_0 \exp(-W/kT)$ is the coefficient of self-diffusion. The number of jogs in the dislocations is determined by $U$, and $E$ is an activation energy for formation of vacancies, due to relaxation of the lattice. The reduction in vacancy formation energy near a dislocation $W$ is estimated from the interaction energy of the climbing dislocations. A comparison of experimental data for Zn and AgBr suggests that $E$ is a significant contribution to the activation energy for ionic crystals but not for metals. The predicted increase in vacancy concentration during creep is too small to give observable effects. It is concluded that any observed enhancement of diffusivity during creep is due to increased mobility of defects.

From author's summary by R. L. Bisplinghoff, USA

**4909. Johnson, A. E., Henderson, J., and Mathur, V., Creep under changing complex stress systems, Parts I, II and III,** *Engineer, Lond.* **206,** 5350, 209–216, Aug. 1958; **206,** 5351, 251–257, Aug. 1958; **206,** 5352, 287–290, Aug. 1958.

In previously published work the authors have established, on an experimental basis, creep-rate stress-time relations which adequately described the behavior of a number of engineering materials at practical working temperatures under conditions of complex stress where the complex stress system remained unaltered throughout the course of creep. However, in engineering practice, in general the stress system causing creep may alter during the course of creep, and it is necessary to generalize the creep-rate-complex stress-time relations to cover such cases. The present work has determined such generalized relations for a 0·2 per cent carbon steel at 450 C, an aluminum alloy RR59 at 150 and 200 C, and a magnesium alloy (2 per cent aluminum) at 20 and 50 C. The experimental data have also been compared with the predictions of the various mechanical theories of creep, and the likely degree of correctness of these latter assessed.

From authors' summary

**4910. Manukyan, M. M., Deformations and stresses in a deflecting ferroconcrete beam while taking into account the creep of the compressed and partially tensioned zone of the concrete** (in Russian), *Izv. Akad. Nauk ArmSSR Fiz.-Matem., Estestv. i Tekhn. Nauk* **9,** 9, 27–45, 1956; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 3430.

An investigation is made of the deformation and stresses in a deflecting beam of rectangular section, having a single bundle of reinforcement, when taking into account the creep of the compressed and, in portions, tensioned zone. The linear theory of succession of N. Kh. Artuyunyan ["Some questions in the theory of creep," Moscow, Gostekhizdat, 1952] is utilized. The hypothesis of plane sections and triangular outline of the stresses is accepted; in the tension zone the stress outline is taken into account on a partial basis, at some height in the region of the neutral axis, and in that portion of it where there are no cracks. Solutions are obtained for the initial momentarily elastic condition, and also for the arbitrary moment of time, with consideration for the creep of the concrete and the increase of the modulus of elasticity. For the solution of the integral equations the method of N. M. Krylov and N. N. Bogolyubov is used. A numerical example is furnished from which it appears that under the influence of creep in the concrete, the deflection is increased by approximately twice, the neutral axis is lowered, the stresses in the concrete are reduced by 47%, while the stresses in the reinforcement increase by 10%. Taking into account the creep of the concrete and the growth of its modulus of elasticity, the stessses in the concrete are reduced by 23% and the stresses in the reinforcement rise by 30%.

I. I. Ulitskii

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4911. Kiiss, I. A., Calculations for ferroconcrete structure when considering the creep and relaxation of concrete** (in Russian), *Trudi Tallinsk. Politekhn. In-ta* **A,** 96, 21 pp. + illus., 1957; *Ref. Zh. Mekh.* no. 11, 1957, Rev. 13427.

A summarized exposition of the bases of a practical evaluation of creep and relaxation, in the sense of their being phenomena of after-effects. The method of introduction of the necessary integral equations into the system of algebraiçal equations was indicated. For the problems determinable by one integral equation, two methods of parallel and nonparallel curves (curves of deformation in creep and curves of stresses in relaxation) are compared; the works of N. Kh. Arutyunyan ["Some questions in the theory of creep," Gostekhizdat, 1952] and of Dischinger [Fr. Dischinger, *Bauingenieur* 1937, (33)–(40)] are made use of. Recommendations are put forward for the evaluation of the accuracy of the approximations in the calculations and how to enhance the accuracy. Application of the bases of the theory is shown in investigations of the appearance of eccentric compression of ferroconcrete elements.

I. K. Snitko

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4912. Bykovskii, V. N., The application of the mechanics of elastic-viscous bodies to the derivation of a theory for the resistance of wood, taking into account the time factor** (in Russian), Investigations on the strength and capacity to deform of wood, Moscow, Gos. Izd.-vo Lit. po Str-vu i Arkhitekture, 1956, 32–41; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 3584.

A type of calculation model is proposed; it has the form of a linearly deforming elastic-viscous body; characteristics are a conditionally-instantaneous modulus of elasticity, a prolonged modulus of elasticity and a period of relaxation, determinable by means of special tests of the wood for tension and compression, and found in practice to be constant with different stresses operating. A sufficiently good conformity was shown between the theoretical curve of deformation aftereffects and the experimental. The conditions were examined governing the transition of the system to the limiting state and the bond was established between the boundary resistance of the wood and the time up to disruption. The influence was demonstrated of the speed of loading on the magnitude of the limit of stability and the limit of plastic flow; it was established that the limit of deformation, in the above circumstances, shows no change.

B. N. Ugolev

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4913. Guard, R. W., Hibbard, W. R., Jr., and Hoffman, R. E., Recent Soviet literature on strength-temperature relation,** *J. Metals* **10,** 11, 727–730, Nov. 1958.

**4914. Noll, D. E., "Thermiatrics," the diseases of high-temperature boilers,** ASME Ann. Meet., New York, N.Y., Nov./Dec. 1958. Pap. 58-A-277, 5 pp.

## Plasticity

**(See also Revs. 4892, 4907, 4908, 4909, 4939, 5001)**

**4915. Ziegler, H., A modification of Prager's hardening rule,** *Quart. Appl. Math.* **17,** 1, 55–65, Apr. 1959.

Prager's hardening rule assumes that the yield surface remains rigid but undergoes a translation in the direction of the strain in-

crement. Author's modification is that the yield surface should still remain rigid and undergo translation, but in the direction of the vector connecting the current center of the yield surface with the stress point. Author maintains that for certain states of stress (e.g. plane stress or combined tension-torsion of a thin cylinder) Prager's hardening rule requires that the yield surface must deform. He claims that his own hardening rule does not suffer from this disadvantage. After a general discussion in tensor mathematics and comparison between the two rules he concludes that his own rule is inferior to Prager's from the physical point of view, but easier to handle in certain classes of application.

Reviewer feels that discussions of this type are too esoteric, especially from the engineer's standpoint, and of limited usefulness. Particularly alarming is the following statement, extracted from the text of the discussion "At the present stage of research, it seems hopeless to expect a decision by experiment between the hardening rules confronted here. Comparisons must be based, therefore, on purely theoretical reasoning." Reviewer believes that the whole purpose of applied mathematics is to describe physical phenomena. If the conclusions of the mathematics are incapable of being put to the test by physical experiment, then the conclusions are valueless other than from an academic viewpoint.

J. M. Alexander, England

**4916. Vakulenko, A. A., The connections between stresses and strains in inelastic media,** *Soviet Phys.-Doklady* **3,** 1, 193–196, Dec. 1958. (Translation of *Doklady Akad. Nauk SSSR* **118,** 4, 665–668, Jan.-Feb. 1958 by Amer. Inst. Phys., New York, N.Y.)

Attempts are being made in many countries to put the stress-strain relations in mechanics of solids on a sound thermodynamic basis. The first and second laws are applied, the free energy function $u\text{-}Ts$ is considered, and (a generalized) Onsage's principle is employed. In this paper the state of a plastic material is supposed determined by stress, temperature, and a set of parameters which will change only when plastic deformation occurs. As is customary, elastic and plastic action are supposed separable. The second law is written $Tds - dq = \psi_{ik}\, d\varepsilon^p_{ik} \geq 0$ where $\psi_{ik}$ depend upon $\eta_{ik}$, the "time" rates of change of plastic strain. The Onsage principle is generalized by taking $\psi_{ik} = \partial\varphi/\partial\eta_{ik}$ where $\varphi = \varphi(\eta_{mn}, T)$ (see also Ziegler in *ZAMP* 1958, pp. 748–763). An explicit result is given for the relation between stress and strain in an initially isotropic medium exhibiting elasticity, viscosity and a thermal "plasticity".

D. C. Drucker, USA

**4917. Muller, H.-G., and Funke, P., Jr., Determination by experiment and by calculation of the rolling force and torque under strip tension** (in German), *Stahl u. Eisen* **78,** 22, 1564–1574, Oct. 1958.

Strip rolling experiments, conducted at the Max-Planck Institute, Düsseldorf, Germany, with strip tension show that the rolling force can best be predicted by the theory presented by D. R. Bland and H. Ford [*Instn. Mech. Engrs. Proc.* **159,** 144–163, 1948] and by H. Ford, F. Ellis and D. R. Bland [*J. Iron Steel Inst.* **168,** 57–72, 1951], when the maximum shear stress at which metal flow starts is taken to be 0.575 times the uniaxial tensile flow stress. The rolling torque can best be predicted by the empirical solutions presented by D. Lueg and E. Greiner [*Stahl u. Eisen* **69,** 879–884, 1949] and by W. C. Hessenberg and R. B. Sims [*J. Iron Steel Inst.* **168,** 155–164, 1951; AMR 4 (1951), Rev. 4185].

D. Kececioglu, USA

**4918. Shchukin, E. D., Pertsov, N. V., and Rozhanskii, V. N., Procedure for the investigation of lack of uniformity in plastic deformation** (in Russian), *Pribory i Tekhn. Eksperimenta* no. 2, 98–102, 1957; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 3607.

A description is furnished of the continuous recording, on a recording oscillograph, of the small oscillations of the test sample with an accuracy of 50 Å with a band pass channel of from 0 to 2000 Hertz and simultaneously of the small changes of resistance in the sample with an accuracy of $0.5 \times 10^{-3}$ Ohm with a band passage channel of from parts of a Hertz up to 1000 Hertz.

From authors' summary

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4919. Onat, E. T., Schumann, W., and Shield, R. T., Design of circular plates for minimum weight** (in English), *ZAMP* **8,** 6, 485–499, Nov. 1957.

Authors use theorems previously established by Drucker and Shield [AMR **11** (1958), Rev. 2125] to solve the problems of the minimum weight design of both sandwich and solid circular plates with edges simply supported or built-in and with any rotationally symmetrical distribution of pressure. As an illustration of the application of the theorems to a structure with body forces, the built-in sandwich plate is solved with the effect of its own weight included; the Tresca yield condition and associated flow rule for an elastic-perfectly plastic material are assumed.

P. C. Dunne, Brazil

**4920. Hawkes, G. A., Residual stresses resulting from the forming of high strength aluminium alloys,** *J. Roy. Aero. Soc.* **63,** 578, 90–94, Feb. 1959.

An X-ray back reflection technique has been used to measure the surface residual stresses resulting from the cold and hot forming of certain high-strength aluminum alloys. The alloys examined were to specifications DTD 683, DTD 687 and B. S. S. L65, and the residual stresses have been related to the residual strain in bending of these alloys. The results show that, apart from the degree of straining, the residual stresses are affected by the heat treatment (cold or hot quench) and the amount (if any) of controlled stretching that the alloy has had between solution treatment and precipitation.

From author's summary

**4921. Brown, N., and Rosenbaum, H. S., The torsional yield stress of zinc single crystals,** *J. Mech. Phys. Solids* **7,** 2, 97–105, Mar. 1959.

The yield stress of circular cylindrical zinc crystals was measured both in torsion and in either shear or tension. Both type of tests were made on the same specimen. The yield stress in torsion varied with the angle between the basal plane and the axis of torsion. When the normal to the basal plane coincides with the axis of torsion, the maximum shear stress in torsion as calculated by the theory of elasticity was found to be $1.6 \pm 0.1$ times the critical resolved shear stress as measured by tension or shear. This value is greater than that found by previous investigators.

If it is assumed that the specimen has slipped almost entirely over its cross section at the macroscopic yield point, then the average stress sustained by the crystal is 1.24 times the critical resolved shear stress.

From authors' summary

## Rods, Beams and Strings

(See also Revs. 4936, 5042)

**4922. Horvay, G., and Mirabal, J. A., The end problem of cylinders,** *J. Appl. Mech.* **25,** 4, 561–570, Dec. 1958.

A rotationally symmetric normal and shear traction distribution which is in self-equilibrium is assumed to act on an end face of a circular cylinder. It is proved that the stresses decay exponentially with the distance from the loaded end. The analysis is based on stress-functions, which satisfy the two stress equilibrium equations identically, and is carried out in a variational ap-

proximation. The result is in agreement with the principle of Saint Venant. L. Foppl, Germany

**4923. Leonov, M. Ya., and Kopeikin, Yu. D., The strength of centrally compressed thin-walled rods** (in Russian), *Nauch. Zap. In-ta Mashinoved. i Avtomatiki Akad. Nauk USSR* **6,** 126–129, 1957; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 3341.

A simplified calculation is furnished for the critical loading for a centrally compressed thin-walled rod of open profile, in the case where there is a slight divergence from complete coincidence of the center of deflection and the center of gravity of the transverse section. It is assumed that the critical force differs from the minimal value of Euler's force $P_y$ when there is present a deflection form of loss of stability, or from the critical force $P_w$ when there is present a torsional form of loss of stability on the relatively small value, for the determination of which V. Z. Vlasov's equation for critical forces is used ["Thin-walled elastic rods," Stroiizdat, 1940]. V. F. Lukovnikov

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4924. Khilobok, G. K., and Khomenko, V. I., A question of investigation of thin-walled rods** (in Russian), *Nauch. Zap. Poltavsk. In-ta Inzh. S.-kh. Str-va* no. 3, 133–147, 1956; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 3340.

A short account is given of the development of the theory of the constrained torsion of thin-walled rods of open profile. A description is also given of two types of apparatus for determining the center of deflection. K. F. Kovalov

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4925. Malter, H., Numerical solutions for beams on elastic foundations,** *Proc. Amer. Soc. Civ. Engrs.* **84,** ST 2 (*J. Struct. Div.*), Pap. 1562, 20 pp., Mar. 1958.

Two methods, following finite difference approaches, for beams on elastic foundations are presented and illustrative examples given. The first method involves successive approximations using correction factors for higher-order differences, and has the handicap of not converging for high foundation moduli. The second method involves writing simultaneous equations based on finite difference approximations to the differential equation. Author deals with the moments and writes equations involving primarily second-order differences. N. M. Newmark, USA

**4926. Hetenyi, M., The infinite beam on equidistant supports and related problems,** Proc. Third U. S. Nat. Congr. Appl. Mech., June 1958; Amer. Soc. Mech. Engrs., 1958, 369–374.

A method of systematic reduction is used in replacing the effect of continuity on the loaded span by two adjoining spans of proper flexural stiffness. The problems analyzed in this manner include:

(a) the infinite beam on equidistant supports,
(b) an infinite sequence of open continuous frames, and
(c) an infinite sequence of closed frames supported at equal intervals.

The procedure is analytical, and the results are "exact" within the limitations of the usual flexural theory of beams.

From author's summary

**4927. Tabakman, H. D., Thin-walled circular beams,** *Mach. Design* **31,** 6, 199–203, Mar. 1959.

Equations and charts are presented for design of hollow structural members which will not collapse under flexure.

From author's summary

# Plates, Shells and Membranes

(See also Revs. 4922, 4950, 4951, 5024, 5048)

**4928. Zaid, M., On the carrying capacity of plates of arbitrary shape and variable fixity under a concentrated load,** *J. Appl. Mech.* **25,** 4, 598–602, Dec. 1958.

The known limit-load solution for a circular plate with a centrally concentrated load is the same for a plate of general shape and fixity and with an arbitrary load location. An ideal plastic material that obeys the Tresca or von Mises condition is assumed. The limit-load based on the square yield condition is essentially greater than that obtained by the Tresca condition. Therefore one should be very careful in the application of the results of the square yield condition. Present paper can be used to establish upper and lower bounds of the limit load for plates under other loading conditions. L. Foppl, Germany

**4929. Kawashima, S., On the vibration of right triangular cantilever plates,** *Mem. Fac. Engng., Kyushu Univ.* **18,** 1, 9–21, July 1958.

Author sets up equations of motion for a vibrating plate in finite difference form and uses matrix iteration to determine modes and frequencies of right triangular cantilevered plates. Calculated values are compared with experimental results obtained by Gustafson, Stokey and Zorowski in 1953 [AMR **7** (1954), Rev. 413]. Agreement was good for first two modes even for a relatively coarse net. Author states that mode shapes calculated were approximately consistent with those obtained experimentally, but comparison is not shown. Method of solving equations and boundary conditions applied are discussed in detail.

E. A. Ripperger, USA

**4930. Wilde, P., The general solution for a rectangular orthotropic plate expressed by double trigonometric series** (in English), *Arch. Mech. Stos.* **10,** 5, 746–754, 1958.

The problem of stability and harmonic vibration of a rectangular orthotropic plate resting on an elastic foundation is considered. To solve this classical problem of the theory of plates the mathematical apparatus of double Fourier transformation is used. The cases under consideration are those of simply supported, free or clamped edges. The possibility of displacement of the comers is taken into consideration.

M. Sokolowski, Poland

**4931. Kaczkowski, Z., Rectangular orthotropic thin plates with arbitrary boundary conditions** (in English), *Arch. Mech. Stos.* **10,** 4, 525–549, 1958.

Author obtains the general solution for an orthotropic rectangular plate loaded in its middle plane by any load. This load may be static or periodic. The solution is represented by a double Fourier series. The boundary values are, in principle, arbitrary with one limiting condition, that of zero vertical displacement of the comers.

The starting point is the displacement equations for plane stress. Author reduces two displacement equations to one differential equation by means of the displacement function. The solution by means of the displacement function is more general than that by means of Airy's stress function because it does not require the assumption that the surface loads have a potential.

The basic layout is the plate with boundary conditions following from assuming the displacement function in the form of a double sine series. The solution for this plate is analogous to Navier's solution for a simply supported plate. To simplify the terminology, the author uses the term of "simply supported plate."

The problem of computation of the expansion coefficients of the stress function for any boundary conditions is reduced to the

solution of a system of algebraic equations. Two numerical examples are given illustrating the application of the method.

Author uses a series of functions which, in general, satisfy neither the differential equation (for instance for $q = 0$ and with loaded edge) nor boundary conditions of the problem. As in his previous works, he starts directly from the differential equation of the problem.

Analogous solutions for thin plates based on the energy method and by means of Lagrange's indeterminate multiplier were obtained by B. Budiansky, R. Hu, and R. W. Conner [Notes on the Lagrangian multiplier method in elastic stability analysis, NACA TN 1558, May 1948; AMR 1 (1948), Rev. 813.] Solution by means of the Fourier transformation was obtained by reviewer.

In the paper reviewed the author abandoned, in principle, the derivation procedure used by him for plates with arbitrary boundary conditions. The solution procedure is rather a generalization of the traditional solution method for clamped plates than a consistent application of double Fourier series.

P. Wilde, Poland

**4932. Mazurkiewicz, Z., Differential equation of equilibrium and vibration and expressions for the boundary conditions of an orthotropic nonhomogeneous plate** (in English), *Arch. Mech. Stos.* **10**, 5, 755–767, 1958.

On the basis of Hamilton's principle the differential equation of equilibrium and transverse vibration of a thin nonhomogeneous orthotropic plate is derived. Using the same method, mathematical expressions for the boundary conditions are obtained. Nonhomogeneous Winklerian foundation is considered.

K. Piszczek, Poland

**4933. Vodicka, V., Contribution to the statics of composite plates,** *Appl. Sci. Res. (A)* **8**, 1, 65–72, 1958.

Author considers elastic cantilever plate-like composite medium formed of infinite strips of different widths and having different material properties. Strips are arranged parallel to one another. First strip has inner edge rigidly clamped to a foundation, second strip inner edge is securely fastened to outer edge of first strip, etc. Solution is found for deflection of each strip when outer edge of last strip is subjected to a concentrated force perpendicular to plate structure. Fourier transform technique is used. Special cases of the two-strip plate and the homogeneous plate strip are discussed, the solutions being given in form of infinite integrals. No computed results are presented.

H. J. Plass, USA

**4934. Choudhury, P., On bending of a circular plate of aeolotropic material under certain non-uniform distribution of load,** *Indian J. Theor. Phys.* **5**, 4, 97–104, Dec. 1957.

Paper is an elaboration of a method used by Timoshenko (1951) and Lekhnitzky (1950). Author uses "engineering" strain definition in terms of derivatives of spatial-displacement, Cauchy's form of the generalized Hooke's law, and finds the stresses induced by a parabolic distribution of loading on one face of the elastically symmetrical plate.

Reviewer's remarks apply to the analyses of the author, Timoshenko and Lekhnitzky. The "engineering" strain dyadic (or second-order symmetric tensor) is unacceptable because it does not transform invariantly with change of axes. The analysis is based on two zero identities between the derivatives of strain: The first is *one* component of the curl of the strain dyadic and the second is *one* component of Saint-Venant's compatibility conditions if the correctly transformable strain dyadic had been used. Reviewer therefore asserts the analysis is invalid and the classical theory is based on unsound premises and, in particular, Saint-Venant's compatibility conditions allow spurious solutions.

K. H. Swainger, England

**4935. Tamate, O., and Shioya, S., On the transverse flexure of a semi-infinite thin plate with a semi-circular notch,** *Bull. JSME* **1**, 4, 357–361, Nov. 1958.

Authors examine stated problem theoretically, starting from well-known biharmonic equation for deflection of a plate. The plate is mapped in curvilinear coordinates and solution obtained by Muskhelishvili's method, using infinite series for the various stresses. A numerical case is evaluated, and authors conclude stress concentration factors are not unlike those around a circular hole in an infinite plate under plain bending, except near corners of notch.

C. E. Turner, England

**4936. Segal', A. I., Cyclic systems in the presence of "rotary loads"** (in Russian), Investigations in the theory of constructions no. 7, Moscow, Gosstroiizdat, 1957, 467–493; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 3149.

Investigations are carried out on a round plate, an infinite plate with a circular orifice and a thin ring during the action of an arbitrary plane balanced system of forces, statically applied to the periphery of the plate or to the axis of the ring. The given loading is presented as an infinite series, each member of which corresponds to an unbroken load of special form, named "rotating" by the author. The vector of its intensity is constant at a value ($p_n$), forming at each point an angle $n\vartheta$ with the direction of the initial radius; here, $\vartheta$ indicates the angular coordinate, $n$ the whole-number index, deriving its values from $-\infty$ to $+\infty$, with the exception of zero (when $n = 0$ the load would be in equilibrium). The "rotating" load at each value of $n$ can be resolved into two cyclic-radial $q_k \cos k\vartheta$ and tangential $t_k \sin k\vartheta$, when the parameters $q_k$, $t_k$ and $p_n$ all become equal, while the indices $k$ and $n$ are differentiated once. The cyclic loading, in its turn, at any value for $k$, can be resolved into two "rotating" entities, one of which will have a positive index and the other, a negative one. Analogously, concepts are introduced regarding "rotating" stresses, acting on the annular and radial sections, on the "rotating" internal forces in the ring and on the "rotating" transfer. The relations are established between $p_n$ and the parameters of the corresponding "rotating" stresses, forces and transfers. When making the deductions the results of the author's works on cyclic problems was taken advantage of. It was shown that the final formulas for "rotating" loads proved to be more compact than those for cyclic systems. Some examples are gone into; the author was able to integrate the series obtained in them. Discussions are introduced dealing with the outlook for developing the method for calculations for a round plate, and also for shells of rotation and three-dimensional rod systems.

Ya. B. L'vin

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4937. Haywood, J. H., and Wilson, L. B., The strain-energy expression for thin elastic shells,** *J. Appl. Mech.* **25**, 4, 546–552, Dec. 1958.

Starting from the linear three-dimensional theory of elasticity for a homogeneous isotropic material, authors derive a general expression for the strain energy assuming (1) that the shell thickness is small compared to the radii of curvature of the middle surface and (2) that the three stress components associated with the direction normal to the middle surface vanish or are negligible. Their general result is then simplified by making the additional assumption, usual in shell theory, that normals to the unstrained middle surface deform without extension into normals to the strained middle surface. The approach used is to expand components of strain and components of displacement relative to the middle surface in powers of the coordinate normal to the middle surface, retaining terms up to second degree in this coordinate. After a discussion of the relative merits of the general

and simplified expressions, special results for a cylindrical shell are compared with those obtained by previous authors.

R. A. Clark, USA

**4938. Borcz, A., Shells stiffened with beams** (in Polish), *Rozprawy Inz.* **6**, 3, 409–430, 1958.

The problem of structural analysis of thin-walled shells reinforced with asymmetric ribs may be solved by introducing three scalar stress functions. The geometrical and mechanical conditions at the point of contact between the shell and the beam are taken into consideration in an accurate manner. In the limit case of a plate the number of the stress functions is reduced to two: the deflection of the plate and Airy's function. The paper is concluded with a numerical example.

M. Sokolowski, Poland

**4939. Derrington, M. G., and Johnson, W., The onset of yield in a thick spherical shell subject to internal pressure and uniform heat flow,** *Appl. Sci. Res. (A)* **7**, 6, 408–420, 1958.

The elastic stresses due to internal pressure and uniform heat flow are quoted separately. In each case, yield occurs first at the inside surface, no matter whether the yield condition adopted be that of Tresca or von Mises. When the shell is subjected to both internal pressure and uniform heat flow, it is shown by the superposition method that yield may occur at any point through the thickness of the shell according to the ratio of internal pressure to internal temperature and the thickness of the shell. However, the effect of temperature on the elastic constants and the yield stress of material is not taken into consideration.

L.-W. Hu, USA

**4940. Anisimova, V. B., Strength of the reinforcing ribs of cylindrical shells** (in Russian), *Nauch. Zap. Kievsk. In-ta* **13**, 8, 5–13, 1954; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 3309.

Investigation of the strength of the reinforcing ribs merges with the investigation of the strength of plane curvolinear rods under the action of an external surface load and of tangential forces, appearing as the result of the interaction of the rib with the sheath, and determinable from the analysis of their co-operative work. Differential equations are deduced for the critical state of a rod of arbitrary outline. An investigation is made of the strength of the compressed and the compressed-distorted rods, when, in the latter case, no regard is taken in the equations of the values of the order of $x^2$ by comparison with $x$ ($x$ being the change of curvature). Problems are solved for (1) the stability of circular ribs reinforcing a closed cylindrical shell loaded by a hydrostatic pressure, (2) the stability of the supporting ring of a pipe conduit. The method of Bubnov-Galerkin is used in both cases for the solution of the basic regulation of the stability. The results of the calculations are given in the form of formulas and tables.

M. S. Kornishin

*Courtesy Referativnyi Zhurnal, USSR*

*Translation, courtesy Ministry of Supply, England*

**4941. Surkin, R. G., Problem of loss of strength of a spherical shell subjected to an external evenly distributed pressure** (in Russian), *Izv. Kazansk. Fil. Akad. Nauk SSSR, Ser. Fiz.-Matem. i Tekhn.* Nauk no. 10, 51–56, 1956; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 3161.

An approximate value is found, by means of the Bubnov-Galerkin method, for the minimum magnitude of external pressure at which a balanced form for the shell is possible, differing from the initial spherical form. A comparison was carried out between the found value of critical pressure and the value of critical pressure obtained by R. G. Surkin by the Ritz-Timoshenko method (1952). Reference is made to the results of the preliminary experiments of R. G. Surkin on the stability of spherical segments under external pressure.

L. I. Balabukh

*Courtesy Referativnyi Zhurnal, USSR*

*Translation, courtesy Ministry of Supply, England*

## Buckling

**4942. Green, A. E., and Spencer, A. J. M., The stability of a circular cylinder under finite extension and torsion,** *J. Math. Phys.* **37**, 4, 316–338, Jan. 1959.

Stability of solid circular cylinder of homogeneous, isotropic, incompressible material in finite extension and torsion is investigated. Analysis is based on general theory of infinitesimal deformations superposed on finite deformations, developed by Green, Rivlin and Shield [AMR **5** (1952), Rev. 2572], and on definition of neutral equilibrium. Formulation of equations is for general strain-energy function; solution is achieved for particular form of this function. Condition of neutral equilibrium is found in form of transcendental determinantal equation. Numerical solution is effected for one non-zero value of extension and for case of vanishing extension. Approximation for the case of small deformations agrees with well-known stability criteria for cylinder under torsion and end thrust.

W. T. Koiter, Holland

**4943. Oze, S. E., Local stability of the walls of steel beams joined to a ferroconcrete plate** (in Russian), *Akad. Nauk Maksimov, Avtomob. Dorogi* no. 10, 25–26, 1956; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 3432.

Some supplementary recommendations are made to standards N and TU, Gushosdora, edition 1948, to meet the case of an unsymmetrical section for the beams, which lead to the use of the known formulas for the critical stresses of compressed-distorted plates.

From author's summary

*Courtesy Referativnyi Zhurnal, USSR*

*Translation, courtesy Ministry of Supply, England*

**4944. Droxdov, P. F., Calculations for steel rods, compressed with initial eccentricities** (in Russian), *Sb. Mosk. Inzh.-Stroit. In-ta.* no. 10, 118–145, 1956; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 3327.

A practical method is given for the calculations for eccentrically compressed steel rods with uneven eccentricities. It is shown that this method of calculation results in an appreciable economy in material compared with the ordinarily used Yasinskii formula for eccentrically compressed rods. The method is illustrated by numerical examples. There are some easily corrected errors and misprints. Analogous results are to be found in Young's work [D. H. Young, ASME and ASCE, Chicago, June, 1933].

Zh. S. Sislyan

*Courtesy Referativnyi Zhurnal, USSR*

*Translation, courtesy Ministry of Supply, England*

**4945. Tsatskis, V. I., Buckling of a rod in a state of compression-tension** (in Russian), Investigations in the theory of constructions no. 7, Moscow, Gosstroiizdat, 1957, 233–238; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 3329.

The critical values are determined for the loading of rods along whose length the normal force changes its sign (indicating the presence of compressed and elongated portions). The rod is examined as a separated element of a plane frame structure in a critical condition during which a buckling of the elements takes place in the plane of structure. The application of the general

equations is illustrated by a selection of special cases. Tables are furnished for the auxiliary functions.

V. M. Makushin

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4946. Petre, A., Thin-walled curved rods** (in Roumanian), *Studii Si Cercetari Mecan. Appl.* **9**, 4, 1013–1021, 1958.

Using the common assumptions made in the study of thin-walled rods, author determines the stress state in curved thin-walled rods of any form of cross section and acted on by a system of any loads. The stress state and the displacement of a cross section are first determined as function of the general deformation. The static conditions then yield the relations between loadings and stresses. By choosing as reducing center a point which plays the part of bending center, general relations are established for calculating the deformations and the stress function of the loading.

Gh. Buzdugan, Roumania

**4947. Peterson, J. P., and Dow, M. B., Compression tests on circular cylinders stiffened longitudinally by closely spaced z-section stringers,** *NASA Memo* 2-12-59L, 22 pp., Mar. 1959.

Six circular cylinders with closely spaced longitudinal stiffeners were loaded to failure in compression. The results obtained are presented and compared with available theoretical results for the buckling of orthotropic cylinders. The results indicate that the large disparity that exists between theory and experiment for unstiffened compression cylinders may be significantly smaller for stiffened cylinders.

From authors' summary

**4948. Kosko, E., The compressive instability of a hydraulic jack,** *Univ. Toronto, Inst. Aerophys. Rep.* 55, 23 pp., Aug. 1958.

After a short discussion of two former approximations author describes a new method of determining the critical load in a pin-jointed hydraulic jack with a cylinder and rod of uniform sections, based on the assumption that the overlapping portions of the cylinder and rod are working independently as cantilever beams. Further it is assumed that the rod is taking the full compressive load over its entire length and that the load transmission through fluid pressure can be neglected. The deflected shapes of the center lines of cylinder and rod segments are obtained; then, by integrating the differential equation of the small-deflection elastic-beam theory, and by eliminating the integration constants through geometric considerations, a resulting equation for the critical load is obtained. This load depends on three parameters and its values are determined from the above equation by an indirect method. Results are presented in eight diagrams covering a wide range of parameters. Discussion of results is followed by a short comparison with former theories and by some remarks on effects of several factors on the critical load. In concluding, author points out the necessity to assess through a systematic series of tests the value of some simplifications, and, in particular, of the assumption that there is no clearance and elasticity at the gland and at the piston.

A. A. Kuhelj, Yugoslavia

**4949. Lisowski, A., Buckling and natural vibrations of cylindrical plates and spherical vaults in the light of model tests** (in Roumanian), *Rev. Construct.* **10**, 10, 474–480, Nov. 1958.

Two assumptions are made by the author: (1) the shape of the deflection curve of a system during buckling is similar to the statical deflection curve of the system under the action of a single force; (2) the shape of the deflection curve in the case of the lowest mode of vibration coincides with the form of the buckling curve. Based on these assumptions, the behavior of cylindrical and spherical shells is analyzed.

Three numerical examples illustrate the theory. The concordance with experimental results is satisfactory.

From author's summary

**4950. Reiss, E. L., Axially symmetric buckling of shallow spherical shells under external pressure,** *J. Appl. Mech.* **25**, 4, 556–560, Dec. 1958.

A simple approximate formula is derived for the initial buckling load for a clamped shell by considering two linearized versions of the nonlinear differential equations for finite deflections. Assuming that constant compressive membrane stresses exist throughout the shell, author obtains a linear homogeneous eigenvalue problem for a deformation function. This problem is solved explicitly in terms of a Bessel function and yields a sequence of critical loads corresponding to different modes of deformation. According to known linear (small deflection) theory, the mode of deformation depends on the size of a certain geometric parameter. As this parameter increases, the mode remains the same until a certain transitional value is reached. The mode then changes to a more complicated form, maintained up to the next transitional value, etc. These transitional values are used to determine the range of the geometric parameter for which a given critical load corresponds to the initial buckling load. The final result is a simple explicit relationship between the buckling load and the geometric parameter. A graph of this relationship shows a certain general agreement with experimental results of Kaplan and Fung [*NACA TN* 3212 (1954)] and with numerical calculations made by the author with Greenburg and Keller [AMR **11** (1958), Rev. 454] based on finite deflection theory.

Recently another paper on the same subject has appeared [Weinitschke, *J. Soc. Indust. Appl. Math.* **6**, 209–322, 1958; also see AMR **12** (1959), Rev. 700]. This gives the results of extensive numerical calculations based on finite deflection theory. The dependence of the buckling load on the geometric parameter appears to be reliably established, at least for moderate values of the parameter. The results differ quite radically from those of the present simplified theory. In the reviewer's opinion still further work must be done before one knows accurately how the theoretical buckling load depends on the geometric parameter.

R. A. Clark, USA

**4951. Moe, H. J., Buckling of shell roofs** (in English), 2nd Symposium on Concrete Shell Roof Construction, Oslo, July 1957; Session II, Pap. 4, 13 pp.

**4952. Langhaar, H. L., and Boresi, A. P., Buckling and post-buckling behaviour of cylindrical shells subjected to external pressure** (in English), 9th Congrès Intern. Mécan. Appl., Univ. Bruxelles, 1957; **7**, 5–7.

**4953. Filin, A. P., On stability of initially stressed structural elements** (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 12, 54–56, Dec. 1957.

The possibility of increasing the critical load for slender structural members by prestressing is shown.

The stability of an element cable of which is in contact with the sheath is considered. Using energy method author deduces the known conclusion concerning the impossibility of buckling during prestressing [G. Magnel, "Prestressed concrete," London 1950, p. 186. A formula for the critical load for the same element charged by an external force is given and discussed. Choosing a corresponding prestress it is possible to assure the stability of the element until its fracture. The formula approaches the influence of creep and shrinkage.

Z. Bychawski, Poland

# Vibrations of Solids

(See also Revs. 4826, 4949, 5016, 5268, 5269, 5285)

**4954. Cox, H. L., A matric formulation of linearly coupled vibration problems,** *Aircr. Engng.* **30,** 353, 202–209, July 1958.

The free and harmonically forced vibrations of complex linear structures that have arbitrarily varying lumped masses and stiffnesses are considered. Lagrange's equations are used for the solution of the mathematical model of the system and the resulting differential equations are then expressed in a convenient matrix notation. The advantage of the matrix formulation is that, in the analysis of problems of this class, the investigator requires only to determine the elements of the matrices representing the spring constants, masses, moments of inertia, coupling coeffients, coordinate transformations and geometry of the structure. The resulting matrix equations are in a form suitable for rapid solution on electronic computers of standard types.

In the opinion of the reviewer, this paper is of great value to engineers and applied mathematicians concerned with the analysis of structural vibration problems in missiles and aircraft structures.

L. A. Pipes, USA

**4955. Jones, R., Torsional vibrational experiments on rectangular concrete beams,** *Brit. J. Appl. Phys.* **10,** 1, 51–52, Jan. 1959.

Earlier anomolous results obtained in measuring the dynamic shear modulus of specimens of concrete by a torsional resonance technique are now explained satisfactorily. A theory, given by Davies in 1938, predicts discrepancies in the apparent shear modulus obtained at higher harmonic resonances due to warping of the rectangular section during vibration. The experimental results are in extremely good agreement with Davies's theory, and it is found that the corrected shear modulus is not significantly influenced by the frequency at which it is measured. This conclusion agrees with the results obtained from torsional resonances on a concrete beam of circular cross section for which no correction is required. From author's summary

**4956. Di Taranto, R. A., Natural frequencies of nonuniform beams on multiple elastic supports,** *J. Appl. Mech.* **25,** 1, 57–63, Mar. 1958.

See AMR **11** (1958), Rev. 1501.

**4957. Protter, M. H., Vibration of a nonhomogeneous membrane,** AFOSR TN 59-279 (Univ. California, Dept. of Math. TR 5; ASTIA AD 212 912), 11 pp., Apr. 1959.

Nehari obtained an inequality between the first eigenvalue of a nonhomogeneous vibrating membrane fixed on the boundary, and that of a homogeneous circular membrane of the same mass. In this paper the first eigenvalues of nonhomogeneous membranes of the same shape are compared. Further, the eigenvalues of two different nonhomogeneous membranes of the same shape are compared. From author's summary

**4958. Kleeman, P. W., The natural frequencies of vibration of triangular cantilever plates of uniform thickness,** *Aero. Res. Lab. Melbourne, Austral. Rep. SM* 262, 25 pp., June 1958.

Problem presented in paper is to determine by large-deflection inextensional plate theory [AMR **9** (1956), Rev. 421] the natural frequencies of triangular cantilever plates for tip angles of 30°, 60° and 90° and a range of sweepback angles. Results for lowest vibration mode obtained by an energy-method solution are compared with small-deflection experimental results of Gustafson et al [AMR **8** (1955), Rev. 1580]. Computed frequency parameters are from 5% to 10% higher than experimental values. No attempt is made to calculate the frequencies of higher modes. Curvature and generator parameters are given to enable the stress distribution in the plate to be calculated. D. F. Muster, USA

**4959. Ul'yunitskii, D. D., Analysis and comparison of experimental modes of free vibration of turbine blades and wide cantilever plates** (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 4, 41–53, Apr. 1958.

Author presents results of a systematic investigation of the natural vibrations of short flat plates, built-in along one edge completely, or over only part of it. The plates tested were rectangular, sectorial, and sectorial with rounded off edges imitating different degrees of idealization of actual hydraulic turbine blades. A comparison of the experimental modes and frequencies of the variously shaped plates indicates a general similarity. Frequencies calculated using simple beam theory and Warburton's method give a fair agreement for the lower modes. A simple formula is given to take into account the effect of a partially built-in edge.

Reviewer believes that the reported investigation is new and is very useful to engineers interested in vibration problems of hydraulic turbine blades, giving a method of estimating general trends and classifying the experimental results.

W. Fiszdon, USA

**4960. Ziemba, S., Vibrations of mechanical systems with one degree of freedom and generalized forces not depending in an explicit manner on time** (in English), *Arch. Mech. Stos.* **10,** 5, 649–669, 1958.

Confining the attention to systems of holonomic-scleronomic mutual constraints, Sec. 1 gives a classification of the elastic and damping characteristics. Author points out the possibility of a great number of combinations of types of characteristics (256) and observes that it is necessary to confine the considerations to those of typical ones. In Sec. 2 the usual method is used to examine the type of the singular points of the initial equation. In Sec. 3 the phase trajectories are investigated by means of the isocline method by comparing their behavior with those of the corresponding equations without damping or the linear equation with damping. In conclusion, the so-called "delta" graphical method is reviewed. Three examples are given for four different types of characteristics. K. Piszczek, Poland

**4961. Lisowski, A., Investigation of shell vibration and stability by means of model tests** (in Polish), *Rozprawy Inz.* **6,** 1, 25–91, 1958.

Paper contains the results of model tests concerning the stability and free vibration of a thin-walled cylindrical, spherical and conoidal surface.

Assuming that the buckling mode of the system is similar to that of deformation under the action of a concentrated force and that the first, basic vibration mode is similar to the form of the same system, criteria of similarity are obtained for the critical load $Q_{kr}/E = \text{const}$, where $E$ is the coefficient of elasticity, and for the vibration frequency $nLM^{1/2}E^{-1/2} = \text{const}$, where $L$ is the characteristic length and $M$ mass per unit volume of the shell. In the final part of the paper a method for model tests is described to verify these criteria. S. Drobot, Poland

**4962. Epshtein, M. O., Determination of frequencies of the natural vibrations of frames, which are not free, by the methods of focus, expressed in a generalized form** (in Russian), Investigations in the theory of constructions no. 7, Moscow, Gosstroiizdat, 1957, 63–85; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 3212.

A general solution is given for the problem of determining the frequencies of the natural deflection vibrations of plane restricted frames by generalized methods of moment and angular foci. The method of moment foci is only capable of determining the frequencies of those forms of vibration in which the amplitudes of the end moments of the rods are not equal to zero. In the general case, when establishing the full frequency spectrum it is essential to make use of both the method of moment foci (the method of forces) and the method of angular foci (the method of deformations). This obstacle is fully met with in the investigation of the strength of frames (the determination of the complete spectrum of critical loads). V. M. Makushin

*Courtesy Referativnyi Zhurnal, USSR Translation, courtesy Ministry of Supply, England*

**4963. Kabulov, V. K., Free vibrations of a girder with a fixed mass at its end** (in Russian), *Izv. Akad. Nauk UzSSR, Ser. Tekhn. Nauk* no. 1, 75–84, 1957; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 3222.

An investigation is made of the free transverse vibrations of a girder carrying on its end a concentrated mass. A comparison is made of the results of the determination of the first frequency by the transcendental equation for frequencies and by an approximate method, based on bringing the girder's mass to its end. This comparison enabled the conclusion to be made that the method of moving the mass permits the calculation of the first frequency of free vibrations of a girder with a concentrated mass at its end with greater exactness. The higher frequencies can not be determined by this method with sufficient accuracy. Formula (3.1) in the paper contains a misprint. N. A. Kil'chevskii

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4964. Vaserman, E. B., Influence of the behaviour of loading on the frequencies of free vibrations of a ring** (in Russian), Questions in dynamics and dynamical strength, 2nd issue; Riga, Akad. Nauk Latv. SSR, 1956, 49–71; *Ref. Zh. Mekh.* no. 11, 1957, Rev. 13109.

Plane and three-dimensional vibrations are investigated, and also the strength of circular rings loaded with an evenly distributed radial load. On the basis of Kirchhof's known equations for a thin curvilinear rod and of Klebsh's kinematic dependencies, equations for frequences are constructed, separately for the vibrations of the ring in its own plane and for the three-dimensional vibrations. Terms enter these equations which depend on the load and which evaluate its behavior in the process of ring deformation during free vibrations. Results were obtained for the three cases out of six not yet investigated: (1) the load remains normal to the undeformed axis of the ring during plane vibrations; (2) the load remains directed towards the center of the initial curvature during plane vibrations; (3) the load remains normal to the undeformed axis of the ring during three-dimensional vibrations. The vibration-frequency values are compared in relation to the behavior of the load. The values for the critical loading of the ring are found. A comparative analysis of the characteristics of plane and three-dimensional vibrations of the ring is carried out in relation to the forms of its vibrations. Round, annular and square sections are examined. D. V. Vainberg

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4965. Beilin, E. A., Natural frequencies of deflection vibrations of arches with elastically fastened abutments** (in Russian), *Nauch. Tr. Leningr. Inzb.-stroit. In-ta* no. 23, 28–42, 1956; *Ref. Zh. Mekh.* no. 11, 1957, Rev. 13111.

An approximate solution is given for the natural frequencies of vibrations of round arches with consideration for the elasticity of its end fastenings, these end fastenings, at the same time, being assumed to be undisplaceable. A precise, transcendental equation is given for the frequencies of the free vibrations of a two-hinged arch, and the unsuitability of such an arch for practical purposes is established. The problem set is solved by the Bubnov-Galerkin method. Girder functions were taken as approximating functions, in which the linear amplitudes were replaced by angular ones. Author asserts that the adoption of these functions permits the determination of the frequency of free vibrations of an arch of any tone. Approximate formulas were obtained, convenient for calculation purposes, for the case of obliquely symmetrical (with two half-waves) and symmetrical (with three half-waves) vibrations of arches. Boundary cases were examined to cover the occurrence of the transition of an elastic fastening to hinged or absolutely rigid fastenings. A. I. Oseled'ko

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

## Wave Motion and Impact in Solids

(See also Revs. 4991, 5318)

**4966. Andrews, E. H., Stress waves and fracture surfaces,** *J. Appl. Phys.* **30,** 5, 740–743, May 1959.

Surface markings, with wavelike characteristics, were observed on the fracture surfaces of frozen carbon-filled rubbers. Interference micrographs support the idea that some of these are "Wallner" lines attributable to interaction of the fracture front with stress waves from the fracture front. It was estimated that the ratio of fracture velocity to wave velocity was about 1:2. Author speculates that the stress-wave oscillations may be caused by a supply of excess energy to the fracture tip.

H. J. Grover, USA

**4967. Mitra, M., Exact transient solution of the buried line source problem for an asymmetric source** (in English), *ZAMP* **9a,** 4, 322–331, Nov. 1958.

Problem considered is that of determining dynamical stresses in a semi-infinite elastic solid due to a line source situated in the interior. Wave function of the source is prescribed; it is purely dilatational in character. The method of Cagniard is used to determine a formal solution for the surface displacement. No numerical work is reported and results are purely of mathematical interest. I. N. Sneddon, Scotland

**4968. Eichler, E. G., and Lambert, R. F., Acoustic (0,1) wave transmission through thin rectangular plates** (in English), *Acustica* **7,** 6, 379–386, 1957.

Transmission loss measurements of thin, rectangular aluminum plates employing the (0,1) acoustic mode of a waveguide are described. Theoretical transmission loss formulas are derived for the limiting cases of small and large ratios of frequency to plate thickness. In the case of large ratios, a "mass law" formulation is obtained which exhibits an angular dependence characteristic of the (0,1) mode. Complete sound transmission predicted for angles near grazing is verified by the experimental results for all plates studied. Some discrepancies between theory and experiment for small ratios are discussed and an attempt is made to correct the calculations empirically. The discrepancies between theory and experiment are here attributed to internal plate dissipation and imperfect boundary clamping.

From authors' summary by M. Harrison, USA

**4969. Seckler, B. D., and Keller, J. B., Geometrical theory of diffraction in inhomogeneous media,** *J. Acoust. Soc. Amer.* **31,** 2, 192–205, Feb. 1959.

Authors provide novel approach to problem of wave propagation in inhomogeneous media at high frequencies. Classical geometrical acoustics leads to the prediction of zero sound energy in shadow zones, a result in disagreement with experience and experiment. In essence, authors provide a means for prediction of the field in shadow zones within the framework of geometrical acoustics. In addition to the rays usually considered in geometrical acoustics they devise diffracted rays which are produced at boundary surfaces and, by analytic continuation, imaginary rays which are produced in refraction regions. These rays comprise the total field in shadow zones. Authors have applied techniques to such problems as propagation of plane waves in a stratified medium and to cylindrical and spherical waves in a stratified medium containing a plane boundary. Authors check their results by comparison with asymptotic solutions of wave theories [see following review].

Reviewer believes that results have important applications in problems such as atmospheric and underwater acoustics.

I. Dyer, USA

**4970. Seckler, B. D., and Keller, J. B., Asymptotic theory of diffraction in inhomogeneous media,** *J. Acoust. Soc. Amer.* **31,** 2, 206–216, Feb. 1959.

Authors find asymptotic solutions of the wave equation for the cases considered in companion paper on geometrical theory of diffraction [see preceding review]. Prime purpose of calculations is to check results of the new theory in the companion paper. Authors use known methods in their calculations. Nevertheless,

apart from its use as a check for companion paper, this paper serves as concise review of high-frequency-wave theoretical diffraction calculations. I. Dyer, USA

**4971. Lessen, M., Thermoelastic waves and thermal shock,** *J. Mech. Phys. Solids* **7,** 2, 77–84, Mar. 1959.

Paper deals with thermoelastic waves and thermal shocks in elastic media. Few investigations have been made in thermoelasticity where account has been taken of the interactions of thermal and elastic effects. Accounting for these forms the feature of the paper. These interactions clearly become important for sudden and high temperatures. Assuming an expansion of the equation of state relating specific internal energy with entropy and strains up to quadratic terms, the momentum and energy equations are reduced to four linear equations for displacements and temperature. The latter differs from the traditional Fourier's equation due to the presence of displacements. These form the basic equations of thermoelasticity. For the isotropic case it is shown that there exist shear waves as in the classical case and two families of longitudinal waves. Formal solutions are obtained for the case of thermal shocks in one dimension, in half-space and infinite cylinder; and it is shown that the thermal effects are limited to a thin boundary layer, while the mechanical effects are dispersed into the medium.

G. A. Nariboli, India

## Soil Mechanics: Fundamental

(See also Revs. 4986, 5317)

**4972. Soeiro, M. F., Habib, M. P., and Tcheng, M. Y., Studies in soil mechanics: suction, thermo-osmosis and surface bearing value of stratified soils** (in French), *Ann. Inst. Tech. Bat. Trav. Publics* **10,** 110, 120–144, Feb. 1957.

In first section (Soeiro) a clear exposition of the theory of suction is followed by some remarks on the relationship between suction and moisture content for soils of different compressibilities, as obtained experimentally by various authors. As an application, the method of predicting equilibrium moisture content of homogeneous and stratified soils under pavements or foundations is also outlined. Results of computations under different hypothesis are presented.

In second section (Habib) the fundamentals of theory of water migration in soils due to a temperature gradient and a summary of laboratory research on this difficult subject are given. Author comments on liaisons between soil and water and on possible relationship with electroosmosis.

In third section (Tcheng) the bearing capacity of a foundation on a shallow sand layer resting on a deep clay deposit is studied by means of small-scale model tests. Author finds that when the thickness of the upper layer is less than 1.5 times the foundation width, the rupture lines in the upper layer are vertical and, as such, have unusual shape. Semiempirical formulas for the computation of bearing capacity based partially on experimental results are given. R. Jappelli, Italy

**4973. Nascimento, U., Contribution for the physical interpretation of soil properties** (in English), 9th Congrès Intern. Mecán. Appl., Univ. Bruxelles, 1957; **4,** 328–342.

Physical laws and concepts are applied to the study and interpretation of soil properties, eight typical Lisbon soils being taken as an experimental basis.

The chief analogies between chemical solutions and clay dispersions are pointed out, the analogy between osmotic pressure and capillary suction being specially analyzed. Herefrom the possibility results of extending the vacuum exsiccator method to determine $\rho^F$ below 4.5.

The connections of the wetting heat with the specific surface and the water/solid interfacial tension are obtained and through them a comparison is made of specific surface values as determined from the wetting heat and from the grain-size distribution; water/solid interfacial tension is also compared with water/air surface tension.

The variation of suction at the water/solid contact film and at the voids of soils are theoretically analyzed and its relations with cohesion and, through experimental results, with Atterberg limits are established.

Terzaghi's equation ($\sigma = \bar{\sigma} + \mu$) is tentatively generalized by subtracting suction from neutral pressure ($\sigma = \bar{\sigma} + \mu - \omega$) and experimental results are presented that justify the continuance of these studies.

Kozeny's formula for permeability is modified, taking into account compacity-induced variations of the hydraulic gradient and the decrease of percolation section due to the motionless water film adsorbed to the grains. From author's summary

**4974. Kofman, V. A., Determination of stresses and deformations arising from the action of a vertical force inside the soil** (in Russian), *Tr. N.-i In-ta Osnovanii i Fundamentov* no. 30, 67–73, 1956; *Ref. Zh. Mekh.* no. 10, 1957, Rev. 12026.

Numerical values for the transpositions and stresses in an elastic semispace, subjected to a vertical force within it, are put forward. The calculations were made on the basis of the exact solution given by Mindlin [*Physics,* **7,** no. 5, 1936] and also developed in the paper by Mindlin and Chen [Mekhanika; Collection of translations and reviews of international periodic literature, no. 4, 118–133, 1952]. The numerical values of the transpositions and stresses, and also the lines of equal transpositions and stresses are reduced with Poisson's coefficient having a value of 0.35. O. Ya. Shekhter

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4975. Vyazemskii, O. V., and Yagodin, G. N., An approximate method for calculating the stability of earth and concrete hydrotechnical installations by means of round-cylindrical and other arbitrary surfaces of slip** (in Russian), *Izv. Vses. N.-i In-ta Gidrotekhn.* **57,** 77–90, 1957; *Ref. Zh. Mekh.* no. 11, 1957, Rev. 13216.

Analyses are put forward for two methods of resolution of forces when calculating the stability of rigid installations and earth slopes by means of round-cylindrical and other surfaces of slip: Terzaghi (direction of the forces of interaction between the soil sections—along the round-cylindrical surface of slip tangential to the cutting) and Kreya (direction of forces of interaction between the soil sections—along the horizontal), where preference is given to the Kreya method. In connection with this the author has worked out a proposal for the evaluation of the forces of soil cohesion. An analysis is also given for a number of calculation cases, and recommendations are made for the practical application of calculations both for rigid installations and for earth slopes.

P. D. Evdokimov

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4976. Berezantsev, V. G., and Yaroshenko, V. A., The strength of sand foundations** (in Russian), *Transp. Str-vo* no. 4, 14–18, 1956; *Ref. Zh. Mekh.* no. 11, 1957, Rev. 13218.

The condition is examined of the limiting equilibrium of sand foundations in different conditions (deepseated foundations and shallow ones with a central loading, with a loading in accordance with the triangular principle, and so forth). The results are given of experiments in the Laboratory of States and Foundations TsNIIS and the Faculty of States and Foundations LIIZhT. The experiments were carried out in tanks with a transparent wall with

stamping dies 2 to 8 cm wide. In making the experiments the recording of the trajectory of the transposition of the particles was done by means of photography and cine-photography. Data are given of the picture of deformation of the foundation and the conditions of formation of a consolidated core. The presence was established of two clearly divided phases of deformation of sand foundations: (1) the consolidation phase, up to the formation of a consolidated core; (2) the phase of significant shearing, after the formation of this core. The possibility of considerably decreasing the dimensions of deep laid foundations was disclosed by the experiments.

Z. V. Pil'gunova

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4977. Evdokimov, P. D., and Lipovtskaya, T. F., Determination of the resistance capacity to shear of cohesive soils in the foundations of structures** (in Russian), *Gidrotekh. Stroit.* no. 11, 43–49, 1956; *Ref. Zh. Mekh.* no. 11, 1957, Rev. 13222.

The results of experiments are given for the determination of the resistance capacity to shear of cohesive soils. The experiments were carried out in a foundation pit by means of displacement by test plates of dimensions 1.75 × 5.25 m and 1 × 3 m and cutting of untouched soil, confined preliminarily in an iron band, and also in the laboratory on cutting appliances. The maximum vertical load was 1.8 kg/cm². It is noteworthy that the stamp shear proceeded along the line of their contact with the foundation, that the relation between the limiting value of the displacement force and the vertical force was practically linear, while the scattering effect on the test points in the field experiments was not significant. The angle of internal friction $\varphi$ and the angle of cohesion $c$ (kg/cm²) was not the same: in stamp displacements $\varphi = 29°$, $c = 0.16$; in cuttings from untouched soil $\varphi = 24°$, $c = 0.3$; in test data using cutting appliances $\varphi = 24°$, $c = 0.36$.

M. V. Malyshev

*Courtesy Referativnyi Zhurnal, USSR*
*Translation courtesy Ministry of Supply, England*

**4978. Maslov, N. N., and Nessler, I. A., The dynamic characteristics of well-rolled sands** (in Russian), 15th Sci. Conf. Leningr. Inzh.-Stroit. In-ta, Leningrad, 1957, 82–84; *Ref. Zh. Mekh.* no. 11, 1957, Rev. 13240.

Data are furnished on the dynamic characteristics of sands of different granulometric compositions and of various degrees of rolling. It is established that the occurrence in the sand of particles less than 0.25 mm in dimensions results in the coefficient of heterogeneity being insufficient for the given purposes. In particular, the actual value of the dynamic stability of sands is contained in the degree of rolling of its granules. The difficulties indicated, in the absence of satisfactory indirect indicators, makes it imperative to establish experimentally a critical acceleration at a given frequency vibration for each of the sands.

Z. V. Pil'gunova

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4979. Kotov, A. I., Deformations of cohesive (clay) soils** (in Russian), *Trudi Novosibirsk. In-ta Inzh. Vodn. Transp.* no. 2, 128–145, 1956; *Ref. Zh. Mekh.* no. 11, 1957, Rev. 13244

Test results are described for compression of artillaceous soils in samples of considerable size (diameter 503 mm) on a specially built device. Lengthy observations were carried out on the deformation of the samples and the interstitial pressures in them by means of hydroaerostatic manometers. Synchronization of the observations every 2–3 secs on the numerous devices in use was obtained by the method of photographic resection. The significant role filled in the general deformation of the sample by creep was noted; this did not damp down in some of the tests for upwards of 6–7 months and was clearly expressed even after only 10–15 hours from the commencement of the tests. The amplitude of the relation of increase of pressure in the interstitial water to the increase of loading on transition to samples with more dense consistency, according to the data of the tests carried out, fell from 0.94 to 0.16. This was again confirmed by the conditionality of the theory of consolidation applicable to soils with a natural structure.

Z. V. Pil'gunova

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4980. Beles, A. A., and Stanculescu, I. I., Thermal treatment as a means of improving the stability of earth masses,** *Geotechnique, Lond.* **8,** 4, 158–165, Dec. 1958.

Paper outlines some characteristic aspects of the available methods for improving the stability of earth masses through thermal treatment. Laboratory test results are discussed, and some of the published data on the technological processes used are described. Details are given on a new procedure of thermal treatment by burning gas oil in borings, using free draught air supply.

Paper includes the description of some practical application of the new thermal treatment procedure as a method for improving the stability of clay soil masses.

From authors' summary

**4981. Bally, R. J., The present state in problems of bearing capacity of earth masses** (in Roumanian), *Hidrotehnica* **3,** 10, 379–386, Oct. 1958.

**4982. Sankaran, K. S., Effect of initial strain on unconfined compressive strength of compacted silty clay** (in English), *J. Instn. Engrs., India* **38,** 12, Part I, 1099–1109, Aug. 1958.

## Processing of Metals and Other Materials

(See also Revs. 4872, 4873, 4874, 4875, 4876, 4917, 4983, 5196)

**Book—4983. Hine, C. R., Machine tools for engineers,** 2nd edition, New York, McGraw-Hill Book Co., Inc., 1959, ix + 445 pp. $7.75.

This book is intended for use with an undergraduate course in the fundamentals of machine tools. It is that, but not much more. It is up to date in the sense that the illustrations, all furnished by machine tool builders, are recent ones. Captions to these figures, unfortunately, often leave something to be desired in completeness. There are a number of errors, such as the statement that high-speed spindles are necessary for internal grinders simply because balancing is too difficult at slow speed. The section on surface finish has a new picture but the text still refers to an instrument long obsolete. A large proportion of the references listed at the end of the chapters are machine tool manufacturers' pamphlets. Despite these limitations the book is still useful because it combines a great deal of information under one cover.

E. G. Loewen, USA

**4984. Rice, W. B., Metal cutting—a research problem,** *Trans. Engng. Inst., Canada* **2,** 4, 149–152, Dec. 1958.

Various analyses of the mechanics of cutting are examined and their predictions compared with experimental data. The development of a more sophisticated model to represent the process is suggested. From author's summary by J. Frisch, USA

**4985. Okushima, K., A sliding-block type cutting test apparatus,** *Bull. JSME* **1,** 4, 338–342, Nov. 1958.

Apparatus for performing machining tests in laboratory is described. Workpiece is moved on sliding block and is cut by fixed

tool, thus creating orthogonal cutting conditions. Force components on tool are measured by dynamometer. Results given for mild steel confirm that cutting force is independent of velocity between 140 and 250 m/min and that specific cutting force decreases with greater depth of cut. Author shows also that shear angle increases with cutting depth.

F. R. Brotzen, USA

**4986. Illei, V., and Kalman, L., Test results on boring method of concrete and clay sheeting** (in Hungarian), *Mélyépitéstudományi Szemle* **7**, 9/10, 300–306, Sept./Oct. 1957.

**4987. Johnson, W., An elementary consideration of some extrusion defects,** *Appl. Sci. Res.* (*A*) **8**, 1, 52–60, 1958.

**4988. Unksov, E. P., and Zavartseva, V. M., Investigation on the stress distributions of a metal when being forged into rollers and bars** (in Russian), *Vestnik Mash.* no. 3, 42–48, 1955; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 3465.

An optical method is used for the investigation of the elastically stressed state in plane and round disks, produced by the action of concentrated and distributed forces in the plane of the disk. When forging and milling of objects from round stock, very often cracks or voids form inside these objects. Authors assume that this disruption is of a brittle nature, and are of the opinion that an investigation of the elastic state may be the answer regarding the conditions giving rise to this crack formation.

K. N. Shevchenko

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4989. Khrapov, M. M., Korolev, A. A., Pobedin, I. S., and Prikhod'ko, I. F., Experimental investigation of the power parameters during the milling of models of deep-flanged beams** (in Russian), Rolling mills no. 8, Moscow, Mashgiz, 1956, 38–54; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 3466.

Measurements were carried out in the Ts NIITMASH laboratory on the laboratory-scale mill of the general and specific pressures and of the necessary torsional moments during the rolling of fashioned objects of deep-flanged beams 75 × 75 mm. An angular dynamometer was used for the measurements. The measurements obtained showed significant divergence from the calculated.

K. N. Shevchenko

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4990. Anisiforov, V. P., Determination of the mean specific pressures in cold rolling without tension, taking into account the elastic compression of the rollers** (in Russian), Rolling mills no. 8, Moscow, Mashgiz, 1956, 195–200; *Ref. Zh. Mekh.* no. 11, 1957, Rev. 13171.

An example is given of the graphical solution of two equations for the determination of the length of the zone of deformation for a metal and for the mean specific pressure during the cold rolling, taking into account the elastic flattening of the rollers. A nomogram is put forward to make the corresponding calculations easier; however it is only of use for the rare cases of cold rolling unaccompanied by tension of the strip.

A. A. Korolev

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4991. Nekrasov, S. S., Influence of rock pressure on the forces required for cutting into brittle materials** (in Russian), *Nauch. Trudi po Vopr. Gorn. dela Mosk. Gorn. In-ta* no. 16, 283–290, 1956; *Ref. Zh. Mekh.* no. 7, 1957, Rev. 8289.

# Fracture (Including Fatigue)

(See also Revs. 4987, 5344)

**4992. Endo, K., and Miyao, Y., Effects of cycle frequency on the corrosion fatigue strength,** *Bull. JSME* **1**, 4, 374–380, Nov. 1958.

Corrosion fatigue was investigated on two carbon steels in the normalized and annealed condition at cycle frequencies of 245 and 2550 cpm in air, tap water and 1% NaCl solution. Stress-cycle (S-N) and stress-time (S-t) diagrams are plotted for 6-mm diam specimens under plane bending and 12-mm diam specimens in rotating bending fatigue.

High on the S-N and S-t diagram the curves are nearly parallel with each frequency, but at stress reversals larger than 2,000,000 or time longer than 30 hours, the curves tend to converge. In any case there is no definite endurance limit.

The notch effect of corrosion-pits produces a stress higher than the nominal value by a factor $k$ which increases with time. Author determines $k$ from the ratio of $\frac{\sigma}{\sigma_c}$ where $\sigma$ is the life-strength in air and $\sigma_c$ is that in the corrosive medium. Experimental formulas are presented to show the influence of corrosion medium, kind of test and time upon the corrosion-stress-concentration factor $k$.

For corrosion fatigue under varying stress amplitudes the cumulative damage criterion followed Minor's rule.

Author should have reported his test curves based upon twenty specimens rather than upon four to ten specimens per curve in order to judge the degree of scatter.

H. Majors, Jr., USA

**4993. Yakubovich, M. A., Studies of endurance limit and impact strength of light ferroconcrete applied in the design of bridges and other structures** (in Russian), *Sb. Tr. Tbilissk. In-ta Inzh. Zh-d. Transp.* **30**, 96–122, 1956; *Ref. Zh. Mekh.* no. 11, 1957, Rev. 13376.

Tests carried out by the author, in co-operation with M. V. Kandelak, on the repeated loading (up to 2 million cycles) showed that the endurance of pumice concrete to compression 60–80 kg/cm² is no lower than the endurance of heavy concrete of a higher grade (~ M-150). In 1952 Sukhum carried out endurance tests on a spongolite ferroconcrete beam with a span of 30 m of concrete grade M-170. With an amplitude of vibrations corresponding to the deflection of the structure due to the working load, the beam without suffering injury (at a frequency of 3–4 Hertz) withstood $2 \times 10^6$ cycles of tests. When the amplitude of vibrations was doubled, fracture took place after 280,000 cycles (jointly with the rupture of the tensed reinforcement in the center of the span). It was established that for automobile road bridges 4 ÷ 17 m made with pumice, tufa and spongolite of grades M-150-170, the dynamic coefficient ensuing from the action of the working load was found to have a mean of 1.2. Test results for span structures for impact are put forward, and also the values of the relative deflections of beam and arch light-ferroconcrete bridges due to the working load. It would seem that the "generalized" formulas recommended by the author can not pretend to claim great accuracy or universality of application, because the formulas do not take into account such factors as creep and shrinkage of light concrete, relaxation of stresses in the reinforcement, etc.

V. V. Pinadzhyan

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4994. Kuldma, Kh. A., Some questions relating to the fatigue endurance of steels when subjected to overstressing** (in Russian), *Trudi Tallinsk. Politekhn. In-ta* **A**, 76, 36 pp. + illus., 1956; *Ref. Zh. Mekh.* no. 11, 1957, Rev. 13552.

Deflection tests were made with test samples of 7.52-mm diameter from bands of steel 30, 45, 30 × and 45 ×; during the tests for

smooth samples overstressing reached $1.5\sigma_{-1}$ and for samples with circular notching $3.0\sigma'_{-1}$. The frequency of the loading varied stepwise with the limits of 5.65 to 50 Hertz. The basic results of the experiments were: (1) The longevity of smooth samples of carbon steel for the whole region of overstressing decreases with the increase of frequency. (2) For smooth samples made from alloy steels at low overstressing the lowest frequency lowers the longevity, while at high overstressing, the effect is reversed. (3) The longevity of the samples with notching decreases with the diminution of the frequency of loading. (4) The indicator of degree $m$, characterizing the incline of the left part of the fatigue curve, with an increase of the frequencies for smooth samples made from carbon steel is lowered, while for samples of alloy steels and notched samples it is increased. (5) For all the steels tested when there is increase of overstressing a tendency appeared for the reduction of the effective coefficient of stress concentration (with large overstressing for steels 30, 45 and 45 × values of $\beta < 1$ and $q < 0$ proved to be possible). (6) With increase of the frequency of loading the effective coefficient of stress concentration decreases. D. I. Gol'tsev

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4995. Argon, A. S., Surface cracks on glass,** *Proc. Roy. Soc. Lond.* (*A*) **250,** 1263, 472-481, Apr. 1959.

Andrade and Tsien [title source **159,** p. 346, 1937] reported finding networks of fine cracks in Pyrex glass which had been etched with sodium vapor. To investigate whether these cracks existed prior to etching, author carried out further tests. These consisted of applying a ball indenter to glass plates and tubes before exposure to the sodium atmosphere. He found that the sodium vapor not only dilates existing cracks but forms extensive new networks. Glasses other than chemical Pyrex were resistant to etching. A. D. Topping, USA

**4996. Argon, A. S., Distribution of cracks on glass surfaces,** *Proc. Roy. Soc. Lond.* (*A*) **250,** 1263, 482-492, Apr. 1959.

The density of cracks in a ball indentation test is assumed to be a function of the Hertzian stress distribution. The integral equation relating the probabilities of fracture, as a function of load, to the crack distribution function is solved with an operational inversion by means of the two-sided Laplace transform. Results are applied to experiments on a chemical Pyrex and a window glass. The latter is found to have more but smaller cracks than the former. A. D. Topping, USA

**4997. Braun, M. P., and Maistrenko, E. E., Tempered brittleness of chromo-manganese-nickel steel in conjunction with its supplementary alloying** (in Russian), *Sb. Nauch. Rabot In-ta Metallofiz. Akad. Nauk USSR* no. 7, 118-126, 1956; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 3526.

The investigation was made on steels of two groups, specially smelted in a 150-Kg induction oven: with a nickel content of 1.3-1.7%, and with a nickel content of 2.3-2.7%. The ingot was forged into small bars at a temperature at the end of the forging operation of 800-850° and the bars were annealed at 850° with cooling in the oven; from the bars test samples of the Menage type were prepared, and these were subjected to different kinds of thermal treatment (hardening and tempering). It is shown that steel 35 X GN subjected to tempered brittleness, after tempering at 550-650° and slowly cooled, loses 43-48% of its viscosity. When the carbon content is reduced to 0.29-0.33 and the manganese content to 1.1-1.2% the sensitivity to tempered brittleness is lowered, whereupon supplementary alloying practically gives no response. Alloying (mixture) W or a combined alloying reduces the tendency of the steel to tempered brittleness. V. M. Kardonskii

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**4998. Finnie, I., The mechanism of erosion of ductile metals,** Proc. Third U. S. Nat. Congr. Appl. Mech., June 1958; Amer. Soc. Mech. Engrs., 1958, 527-532.

The manner in which abrasive particles erode the surface of a ductile metal has been studied analytically and experimentally. By considering the behavior of a single abrasive grain impinging on the surface, equations have been derived which predict the weight loss by erosion. Experimental results for several metals show good agreement with the predicted variation of weight loss with particle velocity and direction relative to the surface. The material properties required for numerical weight loss prediction are shown to be compatible with those obtained from metal cutting experiments. From author's summary

**4999. Kryanin, I. R., Cavitation destruction of blade material** (in Russian), *Metallovedeni i Obrabotka Metallov* no. 12, 9-11, 1956; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 3495.

It is established that the cavitational destruction of propellers is the result of the mutual action of two factors: the mechanical reaction of hydraulic blows, emerging when the cavitational bubbles flap on the surface, and fatigue corrosion, developing owing to the simultaneous reaction of cyclic stresses and river water. As a result intercrystallite cracks are formed.

From author's summary

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5000. Zilova, T. K., Sadovskii, V. E., and Demina, N. I., Influence of the surface state on the tendency of steel Mark ZOKhGSA to delayed rupture** (in Russian), *Metallovedenie i Obrabotka Metallov* no. 8, 2-7, 1957; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 3521.

The fact emerges, from experiments, that mechanical cold hardening, obtained in the process of thread cutting on thermally treated components made of steel Mark ZOKhGSA, increases the sensitivity to notching and retards disruption, in some cases by as much as 1.5 times. From authors' summary

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

## Experimental Stress Analysis

(See also Rev. 5147)

**5001. Barker, A., and Hardy, E. H., Measurement of residual stresses in alloy steel forgings,** *Instn. Mech. Engrs. Proc.* **171,** 17, 581-596, 1957.

Core-trepanning method of Sykes gives mean residual stress, and cylinder-boring method of Sachs gives distribution and magnitude of principal stresses. Owing to time required for machining, accuracy of resistance strain gages over long time is essential. Authors describe in detail the techniques for controlling gage drift for periods of order of months. Principal inaccuracy of measured residual stresses by Sachs method results from inelastic deformation, partially resolved by combination of boring and turning down. Experimental examples are described. Authors estimate accuracy of ±2 ksi if residual stresses is less than 20 ksi; greater percentage error occurs for higher stresses. D. K. Felbeck, USA

**5002. Higson, G. R., Resistance strain gauges of low temperature sensitivity,** *J. Sci. Instrum.* **36,** 4, 157-159, Apr. 1959.

Strain gages manufactured from a series combination of Nichrome (80% Ni-20% Cr) and Advance (60% Cu-40% Ni) wires may be used in quarter-bridge circuits to measure static strains where the temperature varies slowly. The average temperature drift corresponds to 20 lb/in.$^2$ × °C on steel and 2.5 lb/in.$^2$ × °C on dural, and the

sensitivity drop is less than 5% up to 250°C. When used in a full-bridge circuit the drift should be negligible.

From author's summary

**5003. Fessler, H., Centrifugal stresses in turbo-alternator rotors,** Instn. Mech. Engrs., Prepr., 16 pp., 1958.

The frozen stress photoelastic technique was used to determine the peak stresses occurring in rotating discs which represented rotor sections. A large range of shapes of winding slots was tested and a simple empirical relationship was obtained from which the peak stresses at the necks of the teeth can be readily calculated. It was shown that these results are directly applicable to steel rotors.

Semicircular subslots (inward from the winding slots) and interslots (between the winding slots) reduced the peak stresses. The largest possible cooling slots with semicircular roots are recommended.

The stresses in the tips of the teeth due to the thrust of the winding-slot wedges were studied with static models in which the centrifugal force due to the windings was replaced by a tensile force. From these results recommendations were made for the least highly stressed shapes of tooth tips in rotors containing interslots.

From author's summary

**5004. Linge, J. R., Some developments and applications of brittle lacquers—a survey of published work and some original contributions, Part I,** *Aircr. Engng.* **30,** 350, 94–100, Apr. 1958.

**5005. Linge, J. R., Some developments and applications of brittle lacquers—a survey of published work and some original contributions, Part II,** *Aircr. Engng.* **30,** 351, 142–148, May 1958.

**5006. Linge, J. R., Some developments and applications of brittle lacquers—a survey of published work and some original contributions, Part III,** *Aircr. Engng.* **30,** 352, 173–179, June 1958.

## Material Test Techniques

(See also Rev. 5038)

**5007. Nagyvati, B., Experimental determination of tensile strength for rigid materials** (in Hungarian), *Mélyépitéstudományi Szemle* **8,** 7, 300–305, July 1958.

**5008. Pavlov, E. M., Methods of determination of temperature coefficients of moduli of elasticity of thin materials** (in Russian), Avtorefer. Diss. Kand. Tekhn. Nauk Mosk. Vyssh. Tekhn. Uchshche, Moscow, 1956; *Ref. Zh. Mekh.* no. 11, 1957, Rev. 13586.

**5009. Wallquist, G., and Carlen, J.-Ch., The use of the hot torsion test for determining the forgeability of steel** (in Swedish), *Jernkontorets Ann.* **143,** 1, 1–28, 1959.

Torsion tests were made on 20-mm test bars of various steels at temperatures ranging from 700 to 1250 C and with twisting speeds from 10 to 200 rpm. The number of twists to fracture gave a good indication of the relative forgeability of the steels.

From authors' summary

**5010. McDonald, P., Jr., and Edwards, J. A., A tensile impact test for plastics,** Engineering School Bull. North Carolina St. College no. 67, 17 pp., Mar. 1958.

**5011. Yoshizawa, T., and Sekiya, S., On Shore hardness tester (scleroscope) of type D,** *Bull. JSME* **1,** 4, 386–390, Nov. 1958.

**5012. Elonka, S., First step to preventive maintenance—nondestructive testing,** *Power* **102,** 3, 115–137, Mar. 1958.

**5013. McSkimin, H. J., Use of high frequency ultrasound for determining the elastic moduli of small specimens,** *Inst. Radio Engrs. Trans.* **PGUE-5,** 25–43, Aug. 1957.

Many materials, such as single crystals, are available in only small sizes. For specimens having linear dimensions of the order of 2 or 3 mm, a phase comparison technique employing high frequency ultrasonic waves for determining elastic moduli has proved successful. In order to minimize and evaluate the effects of dispersion due to diffraction, transducer coupling and reflection of energy from lateral boundaries, it was found necessary to make measurements over a wide frequency range extending to 200 mc/sec. A description of the experimental methods and of the apparatus used is given, including the construction of units suitable for measurements over a temperature range. Illustrative results for single-crystal indium antimonide are presented.

From author's summary

**5014. McCoy, E. E., Jr., High-pressure test for determining air content of hardened concrete,** Wways. Exp. Sta., U.S. Army, Corps of Engineers, Vicksburg, Miss., Misc. Pap. 6-286, 24 pp. + 1 table + 1 plate + 5 pp. appendix, Oct. 1958.

## Properties of Engineering Materials

(See also Revs. 4893, 4912, 4920, 4995, 4996, 4997, 4998, 5009, 5014, 5037, 5180, 5275, 5331)

**5015. Chechulin, B. B., and Syshchikov, V. I., Cyclical strength of titanium** (in Russian), Metal management, Leningrad, Sudpromgiz, 1957, 196–205; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 3564.

From the authors' experiments it is apparent that technically pure titanium possesses a high endurance limit and a comparatively small sensitivity to incisions during cyclical testing.

From authors' summary

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5016. Gontkevich, V. S., and Butkovskii, V. V., Determination of the loss of energy due to internal friction in the metal during deflection vibrations** (in Russian), *Sb. Trudi Labor. Problem Bystrokhod. Mashin i Mekhanizmov Akad. Nauk USSR* no. 5, 167–178, 1956; *Ref. Zh. Mekh.* no. 7, 1957, Rev. 8468.

Some results are given of the experimental determination of the dissipation of energy in ten different steels when deflection vibrations were dying down. To do so, use was made of the known tuning fork samples, with a recording of the vibration amplitude on a cinefilm made with the aid of a "shleifov" oscillograph MPO-2 and resistance gages, glued to the foot of the tuning fork. The presentation of the experimental results is governed by some premises in regard to the theoretical analysis of the transverse vibrations at the foot of the tuning fork, taking into account the dissipation of energy in the material. In so doing authors proceed from the expression for stresses at a given moment of time in the determined fiber at a distance $\eta$ from the neutral line, proposed by A. P. Filippov:

$$\sigma = E\left[\varepsilon + \frac{1}{w}\left(\mu_0 + \mu_{11}\left|\varepsilon_0\right| + \mu_2\varepsilon_0^2\right)\frac{\partial\varepsilon}{\partial t}\right]$$

where $\mu_0$, $\mu_1$, $\mu_2$ are damping coefficients, $w$ is the vibration frequency, $\varepsilon_0$ is the amplitude of the relative deformation. Coefficients $\mu_0$, $\mu_1$, $\mu_2$ are found from the following equality deduced by the authors

$$\frac{\delta(A_1)}{\pi} = \mu_0 + 6.232\,\mu_1\frac{Cm_1^2}{l^2}A_1 + 35.49\,\mu_2\frac{Dm_1^4}{l^4}A_1^2$$

where $\delta(A_1)$, the logarithmic decrement of the tuning fork's vibrations, is determined on the experimental data on the parts showing reduced amplitudes of from 5-10%.

$$C = \frac{3}{8}b, \quad D = \frac{3}{20}b, \quad A_1 X_1(l) = 0.9744\,y$$

$b$ is the thickness of the foot of the tuning fork, $l$ its length, $m_1$ a characteristic number, $X_1(l)$ a natural function, $y$ the static deflection at the foot's end. The experimental values obtained for the logarithmic decrements of damping by means of the given method for the study of energy dissipation in an extremely heterogenous-stressed condition are presented in the form of curves as functions of maximum stresses at the base of the foot. The coefficients of damping $\mu_0$, $\mu_1$, $\mu_2$ found in this way are given in a table. Paper includes confirmations of such earlier established facts, as, for instance, the growth of the logarithmic decrement with the increase of amplitude, and its independence from the frequency (in the limits of 100-625 Hertz). It is shown that the coefficients of damping thus obtained may be utilized for the calculation of the transverse vibrations of rods of arbitrary form and with arbitrary aspect of the stressed condition.

G. S. Pisarenko

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5017. Lynboshits, M. I., Influence of the speed of step-wise loading on the mechanical characteristics of the strength of oak timber** (in Russian), *Sb. Nauch. Rabot. Belorus. Politekhn. In-ta* no. 54, 156-166, 1956; *Ref. Zh. Mekh.* no. 10, 1957, Rev. 12281.

Tests for deflection in the radial direction were carried out on samples of air-dry oak (moisture content 10.7-11.3%). The samples were 26 × 20 mm in section with a span of 400 mm; the load was applied in equal stages, the samples being kept under load at each stage. The limit of the plastic yield $\sigma_{py}$ was determined. When the duration of the test was increased from 8.6 to 9.5 minutes to 21.9 days (24-hr) the limit of strength $\sigma_s$ dropped from 904-922 to 752 kg/cm², while $\sigma_{py}$ from 486-488 to 382 kg/cm²; $\sigma_{py}$ dropped faster than $\sigma_s$. When the rate of loading is reduced, deformation for $\sigma_{py}$ increases significantly.

Yu. M. Ivanov

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5018. Giles, C. G., and Sabey, Barbara E., Rubber hysteresis and skidding resistance,** *Engineering* **186**, 4842, 840-842, Dec. 1958.

The dependence of skidding resistance of automobiles on temperature is discussed. Friction measurements on various surfaces at temperatures between 0 and 40 C all show a falling off in slide resistance with rising temperature. The skid resistance of different types of rubber are compared with the rebound tests. Rubber giving higher rebounds showed lower skid resistance. Except in the case of the rubber giving the lowest rebound, friction fell off with increasing temperature. The rubber giving the lowest rebound gave a slightly increasing friction as temperature was raised. (It is to be noted that the captions of Figs. 3 and 4 appear to have been interchanged). Full-scale tests on various road surfaces are described, as well as the effects of tire tread pattern on drainage under wet conditions.

F. T. Barwell, England

**5019. Cassie, A. B. D., The physics of fibres with special reference to wool,** *Brit. J. Appl. Phys.* **9**, 9, 341-348, Sept. 1958.

There are many angles from which one could approach the physical study of fibrous materials. In the book under a similar title published in the Physics in Industry series, Woods has dealt largely, but not exclusively, with those aspects of the study which are concerned with the consequences of the molecular structure of fibrous substances. On the other hand, there are many interesting physical problems, particularly in relation to fiber assemblies such as fabrics, which arise purely from the geometrical form of the fibers and their ability to exchange water vapor with the atmosphere. This paper is largely concerned with these aspects of the subject.

From author's summary

**5020. Sippel, A., and Hutzen, E.-H., The dependence of the tensile strength of filaments on its length between the jaws. Part II. Experimental** (in German), *Faserforsch. u. Textiltech.* **9**, 6, 213-217, June 1958.

Results from experimental tests of the Weibull theory of the tensile strength with endless fibers from acetate rayon are communicated. They show that this theory is not applicable to endless artificial fibers. The evaluation of the measurements leads to an empirical relation between the tensile strength and the length between the jaws. Thus the tensile strength may be calculated for the length between the jaws of zero and infinite values; the last constitutes a considerable fraction of the first.

From authors' summary

**5021. Otsuki, S., On the strength of the glass fiber reinforced polyester (damage of the glass-cloth by pressing and influence of moisture and dryness),** *Bull. JSME* **1**, 3, 244-250, Aug. 1958.

The tensile strength of the glass-fiber-reinforced polyester (FRP) increases by the glass content by volume in the linear relationship. However, it decreases when the glass content exceeds a certain critical point. According to the consideration that this is caused by the damaged glass fiber in the pressing procedure for forming, the tensile strength of the glass cloth was examined after being pressed by a known pressure, and it was revealed that the strength was reduced suddenly beyond a certain critical pressure. This pressing pressure coincides with the forming pressure of FRP at its critical tensile strength, which leads us to the conclusion that the chief factor for this phenomenon should be the damage of glass fiber due to the forming pressure of FRP. In addition, by the experiments for dried and wet specimen, it was clarified that the critical pressure is higher for dried specimen than the wet, the strength is reduced by moisture not only for alkali glass fiber, but also nonalkali ones, and that the strength in wet condition is restored by heating or redrying. This was explained by means of design of experiment.

From author's summary

**5022. Vasil'kov, A. N., The strength of concrete and other stone materials** (in Russian), *Nauch. Tr. Kazan. In-ta Inzhenerov-Stroit. Neft. Prom-sti* no. 2, 3-70, 1954; *Ref. Zh. Mekh.* no. 11, 1957, Rev. 13591.

Starting from the assumption that in dispersion materials the destruction of slip is accompanied by fracture, author reaches the conclusion that the so-called "plane" of slip is really a surface consisting of separate ledges or "teeth," one side of which undergoes slip, and another side fracture. Formulas are obtained for the determination of the prismatic and cube stability of concrete under compression, for strength of clean shear and for conditions for the stability of concrete and stones when in complex stressed conditions $\sigma_1 > \sigma_2 = \sigma_3$, $|\sigma_3| > |\sigma_1| = |\sigma_2|$. A graphical method and mechanical apparatus were constructed for the determination of the limits for $\sigma$. The results obtained from calculations are supported by numerous data of various authors and are illustrated by detailed examples (concrete in bands, stonework with skew reinforcement, etc.).

G. S. Grigoryan

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5023. Linkov, E. M., Elastic ice characteristics and methods of studying them** (in Russian), *Vestnik Leningrad Univ.* no. 16, 57-68, 1957.

Methods of studying elastic properties of ice are discussed. Results of measuring elastic moduli of ice by static, resonance, ultrasonic and seismic methods are given. The analysis of these data and peculiarities of the methods are the basis for the conclusion that the seismic method of investigation is rather perspective, especially in the case of studying elastic characteristics of large heaps of ice (glaciers, ice domes and islands).

It is pointed out that the influence of meteorological conditions on the elastic properties of ice bodies is studied insufficiently; on the other hand, there are no data concerning dependence of elastic characteristics upon the peculiarities of ice-body structure, which limits practical application of the known information about elastic characteristics of ice. From author's summary

## Structures: Simple

(See also Revs. 4911, 4965, 4993)

**5024. Drucker, D. C., and Shield, R. T., Design for minimum weight** (in English), 9th Congrès Intern. Mécan. Appl., Univ. Bruxelles, 1957; **5,** 212–222.

Problem is to find, within some space limitation on the extent of the structure, the minimum weight, i.e. the minimum volume of a structure of homogeneous material designed to carry a given distribution of loads. The material is idealized as perfectly plastic. Whereas, till now, the problem of minimum weight of structures has been solved for special cases, e.g. frames and plates [AMR **6,** (1953), Rev. 2787; **7,** (1954), Rev. 768; **8,** (1955), Rev. 75; **9,** (1956), Revs. 1095, 2157], in this paper a general result is found.

From the main theorems of limit analysis for continuous media [AMR **5,** (1952), Rev. 1695], giving upper and lower bounds on the minimum volume and by comparing the rate of work done by the external forces and the internal dissipation in a structure just at the point of collapse with the corresponding work and internal dissipation in any other structure which is safe or just nearly at the point of collapse, the following general conclusion results:

Plastic strain-rate fields for which $D$, the rate of dissipation of energy per unit volume, is constant give minimum weight designs. For space structures, sheets, and membranes an absolute minimum is obtained with $D$ a constant throughout the permissible volume. For beams and plates, a relative minimum is found for $D$ a constant at the surfaces of the plate.

Fields of constant rate of dissipation do not generally exist and, except in special instances, cannot easily be found when they do. In a later paper author intends to consider means of overcoming this difficulty. For a particularly simple example—the circular annular disk loaded by uniform forces on the inside and on the outside—solutions are given.

Paper is a valuable contribution to the problem of designing structures of minimum weight. E. Seydel, Germany

**5025. Bechert, H., The design of orthogonal load groups with help of Schmidt's method of orthogonalization** (in German), *Bautechnik* **34,** 3, 83–86, Mar. 1957.

A brief description of Schmidt's method of orthogonalization is given. Applying method to statically indeterminate structures results in shortened calculations. Examples given show application to frames and continuous beams. W. D. Jordan, USA

**5026. Dalban, C., Behaviour and structural analysis of steel-frame pillars in the elastic-plastic range** (in Rumanian), *Rev. Construct.* **10,** 12, 585–593, Dec. 1958.

Paper summarizes the present stage of studies undertaken by different authors regarding the behavior and structural analysis of steel frame columns in the elastic-plastic range. The case of pillars of industrial halls, where the ultimate stage method of calculation is used extensively, is emphasized. Conclusions are drawn and theoretical and practical indications are given.

From author's summary

**5027. Johnson, L. P., Jr., and Sawyer, H. A., Jr., Elasti-plastic analysis of continuous frames and beams,** *Proc. Amer. Soc. Civ. Engrs.* **84,** ST 8 (*J. Struct. Div.*), Pap. 1879, 23 pp., Dec. 1958.

Authors present a method by which a continuous beam or frame loaded to plastic range can be analyzed. The method involves a process of successive approximation, similar in many respects to Cross's moment-distribution method. A statically permissible set of moments is first assumed, then the corresponding plastic deformations are evaluated, followed by the calculation of moments responsible for such plastic deformations. The computed moments are then compared with the assumed moments and the process will be repeated until the computed and the assumed moments become close. Several examples are presented for illustration.

D. H. Cheng, USA

**5028. Tung, C. P. C., Design of pier bent and rigid frame by a computer,** *Proc. Amer. Soc. Civ. Engrs.* **84,** ST 7, (*J. Struct. Div.*), Pap. 1854, 17 pp., Nov. 1958.

This paper discusses the basic approach and procedure of using moment distribution in the design analysis of a pier bent or rigid frame by electronic computation. Individual effects of standard loadings can be automatically combined to give the maximum design values. Sample solutions from the completed program, coded for the LGP-30, are illustrated. The potentialities of solving similar structural problems employing the same general principle are explored. From author's summary

**5029. Kandov, I. B., Method of successive approximation in investigating the elastic stability of plane frame systems** (in Russian), *Sb. Nauchno-issled. Rabot Tashkentsk. Tekstil'n. In-ta* no. 3, 101–109, 1956; *Ref. Zh. Mekh.* no. 11, 1957, Rev. 13276.

The right side of the differential equation for the longitudinal-transfer deflection of a rod of constant section $By'' = -Py + f(x)$ is replaced by a stepped function $K(x/l)^n$, the region of application of which is established. For the evaluation of the critical condition of the system the bases of the graphoanalytical method are made use of. In the example given for the calculation of an indisplaceable two-span two-storied frame three full solutions for the frame had to be carried out by the method of the distribution of moments. I. K. Snitko

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5030. Moliotis, P. D., Generalization of the successive approximations method,** *Proc. Amer. Soc. Civ. Engrs.* **84,** ST 6 (*J. Struct. Div.*), Pap. 1816, 35 pp., Oct. 1958.

Paper presents a method for the analysis of rigid frames, where the joints are assumed to be small undeformed areas. First part, termed "stress analysis in two stages," parallels moment distribution procedure with multiple sidesway corrections. Author develops formulas for fixed-end moments and shears; distribution and carry-over factors; fixing forces in the imaginary supports; and moments due to arbitrary movements in the direction of the imaginary supports for members rigidly encased at their ends and for joints where the instantaneous center of rotation is not at the center of the joint. In second part, termed "stress analysis in one stage," after successive joints are allowed to rotate, successive imaginary supports are removed and the change in moments computed. Author develops formulas for the relationship between unbalanced moment at any joint and the fixing force in any imaginary support; the change in moment in any member due to the removal of any imaginary support; and the additional fixing forces in the imaginary supports due to the removal of one support. Examples of use of both methods are given. S. J. Fenves, USA

**5031. Tuma, J. J., Havner, K. S., and Hedges, F., Analysis of frames with curved and bent members,** *Proc. Amer. Soc. Civ. Engrs.* **84,** ST 5 (*J. Struct. Div.*), Part I, Pap. 1764, 32 pp., Sept. 1958.

A modified moment-distribution method is presented for the analysis of continuous plane frames with curved and bent members. A new structural parameter, the moment-thrust factor, is introduced. The possibility of computing the moments due to rotation by the sum of infinite series is demonstrated. Numerical studies are included.

From authors' summary by M. Botman, Canada

**5032. Trysna, F., Two-panel linked beams and roof trusses with unequal spans** (in German), *Bautechnik* **34,** 3, 86–88, Mar. 1957.

Continuous beams of two unequal spans with different cross sections for the two spans are considered. Adjacent beam sections are lap-jointed over the center support. Author concludes that no significant economy is gained in this case over that of a single beam used over both spans. W. D. Jordan, USA

**5033. Samelson, H., and Tor, A., Stresses in reinforced concrete sections subject to transient temperature gradients,** *J. Amer. Concr. Inst.* **30,** 3, 377–386, Sept. 1958.

Authors evaluate by finite differences the successive temperature distributions across the thickness of relatively thick reinforced-concrete sections, developed during temperature changes of relatively long duration. Then they take the thermal stresses proportional to the deviation from the average temperature across the thickness of the cylindrical tanks considered. It is found that, for low conductivity and time-dependent temperatures of the liquid inside the cylinder between 50 and 500 degrees Fahrenheit, the stresses at some transient stage are considerably higher than the ones reached during the final steady-state linear temperature profile. G. A. Zizicas, USA

**5034. Pishchik, G. F., A question regarding the influence of the stressed state of the concrete on the stressed state of the reinforcement** (in Russian), *Trudî Novosibirsk. Inzh.-Stroit. In-ta* no. 5, 175–185, 1955; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 3444.

Models prepared from Bakelite reinforced with aluminum were used. The stresses in the Bakelite were determined by the optical method, while for the stresses in the reinforcement theoretical calculations were employed. Zh. S. Sislyan

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5035. Abeles, P. W., Partially prestressed construction (theory and design)** (in German), *Bauingenieur* **33,** 3, 77–84, Mar. 1958.

**5036. Haviar, G., Analysis of posttensioned concrete beam ends** (in Hungarian), *Mélyépítestudományi Szemle* **8,** 4/5, 165–170, Apr.-May 1958.

**5037. Vishnevetskii, G. D., Introduction to the technological theory of the shrinkage and swelling of concrete** (in Russian), 15th Sci. Conf. Leningrad Engng.-Constr. Inst., Leningrad, 1957, 356–363; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 3582.

It is assumed that the concrete body is homogeneous in regard to the processes of hydration, diffusion, humidity and thermal conductivity, isotropic and quasi-homogenized, porously solid. The physico-chemical state of the concrete is described with the aid of four parameters: temperature, extent of firmly-bound water, extent of weakly-bound water and extent of free water. The last three parameters are expressed as "parts" in the water-cement relationship. The shrinkage is expressed as the linear ratio of the difference between the initial and final water-cement relations. The expressions for the calculated values are recorded in a general form. The equations contain many coefficients, the methods for their determination not being disclosed.

A. E. Desov

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5038. Ginzburg, Ts. G., Laboratory tests on the water-impermeability and stability of block seams of concrete structures** (in Russian), *Izv. Vses. N.-i. In-ta Gidrotekhn.* **56,** 144–155, 1956; *Ref. Zh. Mekh.* no. 11, 1957, Rev. 13434.

Five types of block seams were subjected to stability tests. The following deductions were made: (1) the incizing of the surface of the concrete strongly diminishes the stability of the seam; (2) the same sort of incision but with subsequent rubbing in of a cement solution significantly increases the cohesion of concretes of equal growth; (3) vibration treatment of the concrete increases its strength by comparison with plugging by 42% on average.

M. M. Manukyan

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5039. Laushey, L. M., Direct design of optimum indeterminate trusses,** *Proc. Amer. Soc. Civ. Engrs.* **84,** ST 8 (*J. Struct. Div.*), Pap. 1867, 35 pp., Dec. 1958.

A method is proposed for the direct design of indeterminate trusses. The principle of potential work is introduced to obtain the maximum compatible stresses for the bars. Redundants are selected to yield the minimum weight of truss. The final bar areas follow directly by dividing the forces in static equilibrium by the stresses satisfying continuity. The relative weights of alternative structures and the optimum structure are revealed by the direct-design method. From author's summary

**5040. Schmidt, L. C., Fully-stressed design of elastic redundant trusses under alternative load systems,** *Austral. J. Appl. Sci.* **9,** 4, 337–348, Dec. 1958.

Direct design of elastic redundant trusses subjected to a single load system is not new [AMR **7** (1954), Rev. 72; L. M. Laushey, *Proc. Amer. Soc. Civ. Engrs.* **84,** ST 8 (*J. Struct. Div.*), Pap. 1867, Dec. 1958, especially discussions]. Minimum weight obtains when the stress is made statically determinate by suppressing appropriate redundant members [AMR **7** (1954), Rev. 3541]. Merit of paper lies in extension to trusses subject to more than one load system. Author assumes that minimum weight is attained when each member is fully stressed under at least one load system. He proposes two iteration methods for obtaining such a solution; the first is to be used until it is known which load system fully stresses each member, turning then to the rapidly converging second method. Paper includes illustrative examples. Methods are described only for the case when the same unit stress governs design of all members. But there is no difficulty in extending it to cover unequal allowable stresses in tension and compression.

E. Rosenblueth, Mexico

**5041. Malter, H., Numerical solution for interconnected bridge girders,** *Proc. Amer. Soc. Civ. Engrs.* **84,** ST 6 (*J. Struct. Div.*), Pap. 1815, 19 pp., Oct. 1958.

Paper is concerned with approximate calculation of deflections and bending moments in girders of simply supported, single span bridges in which girders are interconnected either by a deck slab or by a set of diaphragms. Symmetrical two-girder and four-girder structures under transversely antisymmetrical loading are considered. Formulas for mid-span girder moments and deflections are obtained by lumping distributed interconnecting elements into single transverse member at the mid-span or transverse members at quarter points, and by assuming girders to be infinitely stiff in torsion. A solution is also obtained by finite differences using four intervals and assuming interconnections to be distributed continu-

ously. Results are compared with those presented previously by Pippard ["*Studies in Elastic Structures,*" London, Arnold, 1952] and are thus shown to be sufficiently good for engineering purposes over a reasonable range of transverse stiffness if the assumption of infinite torsional rigidity of the girders is admissible.

J. E. Goldberg, USA

**5042. Stupishin, N. A., Adaptation of the method of successive approximations to the investigations of the strength of rods and arches** (in Russian), *Trudî Mosk. In-ta Inzh. Zh.-d. Transp.* no. 92/11, 122–136, 1957; *Ref. Zh. Mekh.* no. 11, 1957, Rev. 13270.

Two methods of calculation are proposed, differing from the ordinary ones in that they give a two-sided approximation to the critical loading being sought. For one of the calculations the deflections of the first approximation are sorted out, with fixed transformations of points of application of external forces, into those known to have large actual deflections, and for the second, those known to have small ones. Examples are given of calculations for a rod of variable section, for a double-hinged round arch of constant section; and the same with varying grades of section.

A. A. Pikovskii

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5043. Kozlovskaya, M. A., Calculations for elastic arches in their deformed condition,** 15th Sci. Conf., Leningrad Engng.-Struct. In-ta, Leningrad, 1957, 371–380; *Ref. Zh. Mekh.* no. 11, 1957, Rev. 13266.

A method is examined of calculations for deformed arches, based on the solutions of Euler's equation, which is built up for a function expressing the potential energy of the system. The inaccuracies tolerated and the unjustified simplifications deprive the work of scientific value. The extensive literature on the problem is ignored.

A. A. Pikovskii

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5044. Csonka, P., Alteration of Barta's iteration method used for determination of critical pressures** (in Hungarian), *Magyar Tud. Akad. Musz. Tud. Oszt. Köz.* **21,** 1/4, 362–364, 1957.

**5045. Muzychenko, Yu. N., Strength of rod assemblies, regular in one direction** (in Russian), *Trudî Rostovsk.-n/D. Inzh.-Stroit. In-ta* no. 5, 189–200, 1956; *Ref. Zh. Mekh.* no. 11, 1957, Rev. 13334.

The strength is examined of a rod system compressed in two directions. The rigidity of the rods in direction $x$ is taken to be equal; the rigidity of the rods in another direction varies in transition from one level to another; the loading in this direction varies from level to level; the compression load on the rods in direction $x$ is taken to be constant for each bar and changes with change of level. The solution is built up by the method of transformations and on the assumption that the angles turn relative to axes $x$ and $y$, and that the transfer of joints of a level conform to the principle of sines. The results establish the fact that the examination of the strength of rod assemblies can be combined with the check of the strength of a single-braced rod system, loaded in the same way as are the braces in a multi-walled system.

A. V. Karmishin

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5046. Bol'shakov, K. P., Lowering of stress concentration in the joints of welded span constructions** (in Russian), *Trudî Vses. N.-i. In-ta Transp. Str-va* no. 20, 6–85, 1956; *Ref. Zh. Mekh.* no. 11, 1957, Rev. 13398.

Results of investigations are given for the scrutiny of constructional and technical measures taken to lower stress concentrations in some of the components of welded span constructions of railway bridges, particularly in shaped joints, lapwelded to the booms, and also in joints in which the angles of union are lapwelded to the horizontal rigid rib, without making use of shaping. Experiments were carried out with test samples of steel Mark St. 3 smelted by the rimmed or killed steel process, in conformity with the comparative tests for durability. It was established that the effective raising of fatigue endurance of the joints, lapwelded and with shaped horizontal unions, was attained by the compulsory removal of all factors inducing stress concentration: the connection of shaped joints to the boom by oblique or flanged seams, the use of mechanical treatment of seams at their point of union with the basic metals, with a spherical shaped cutter or abrasive wheel, etc. It is shown that the making of joints for the horizontal unions with direct fastening of the angles of the diagonal to the rib, in the absence of seam treatment, is irrational. The mechanical treatment of the edges of the angular seams raises effectively the durability of the structure. The durability does not change appreciably with variations in the sequence of superposition of the seams.

G. A. Nikolaev

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5047. Farkas, M., Calculation of maximal moments** (in Hungarian), *Mélyépítéstudományi Szemle* **8,** 4/5, 199–201, Apr.-May 1958.

**5048. Schnause, R., Investigation of a fireproof sandwich construction** (in German), *ZVDI* **100,** 12, 519–521, Apr. 1958.

In ship-building, the material for cabin wall and deck are required to be fire-proof by the marine insurance agreement contracted at Seattle in 1948. To find material satisfying the requirement, some sandwich-constructed boards were tested with the fire-chamber method enacted by the agreement. The following four samples, 3/4 in. in thickness, were tested; three of them were "internit" (3.2-mm plate made of cement, cellulose, asbestos)-boards filled with (a) rock wool, (b) diatomaceous earth, (c) foamed concrete, and the fourth, (d), "eternit" (cement, asbestos)-board filled with rock wool. It was shown by the test that (d) is best and (a) is poor, (b) and (c) have the tendency to be locally overheated through the chinks in the filling caused by vibration. Author prefers "internit" to "eternit" because the former (its density is 0.67 g/cc) is lighter than the latter (0.92 g/cc) and even the sample (a) comes up to the standard.

Then four "internit"-boards with rock wool fillings, thin and thick, with rock wool filling and air spaces, and with air space only, were tested. The result shows that the heavier a board is, the more fire-proofing power it has. Author concludes that the fire-proofing power of material increases its value more rapidly than the weight of material and consequently than the cost of freighting.

I. Sawai, Japan

**5049. Wansleben, F., Critical thoughts concerning the design of cross-connected multiple pressed rods** (in German), *Stahlbau* **27,** 1, 15–16, Jan. 1958.

**5050. Owada, S., Elastic properties of an orthogonally woven structure,** *Trans. Japan Soc. Aero. Engng.* **1,** 1, 19–25, 1958.

Wire mesh or weave is treated as orthotropic structure under one- or two-dimensional tension loads. Bending and direct strains are taken into account as well as deformation of the members under contact pressure. Center lines of members are assumed sinusoidal. The median plane of the structure is assumed to remain undeflected. Friction between members at points of contact is neglected.

Maximum stresses, effective extensional stiffnesses and Poisson's ratios are given for various types of loading. Maximum

stress does not necessarily occur at contact point and Poisson's ratio may exceed 0.5.

Comparison with tests on steel wire mesh shows good agreement for stress and stiffness but no correlation for breaking loads, due to the assumption of purely elastic deformation in the theory.

Reviewer believes that the proposed extension to glass-fiber weaves is questionable because of their different type of deformation under contact pressure. E. Pribram, England

**5051. Obraztsov, I. F., Deflection and torsion of a multiply-fastened multi-web construction** (in Russian), *Trudi Mosk. Aviats. In-ta* no. 86, 67 pp. + illus., 1957; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 3174.

Calculations are described for a four-contour multiweb construction subjected to deflection and torsion, taking into account auxiliary stresses arising in consequence of constrained deplanation; the influence is shown of some geometrical dimensions on the stressed state of the shell. The results of the experiment are given. From author's summary

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

## Structures: Composite

(See Rev. 5322)

## Machine Elements and Machine Design

(See also Revs. 4847, 4848, 4849)

**5052. Freudenstein, F., Structural error analysis in plane kinematic synthesis,** *ASME Trans.* **81B** *(J. Engng. Industry),* **1,** 15–22, Feb. 1959.

Author has made another valuable contribution to the subject of kinematic synthesis by presenting a procedure for finding a linkage that will be the best approximation of a given function.

Based on Chebychev's approximation theory, author derives the method of modified Chebychev spacing for the preselection of precision points in the analytic synthesis of function generation. He adapts the Chebychev mathematical approach to the digital computer, thus making possible the use of modern facilities for carrying out otherwise burdensome and lengthy calculations. It also opens the way to investigation which otherwise would have been difficult to undertake because of the complexities involved.

Of interest also is the suggested use of the method to chart generating mechanisms. M. Martellotti, USA

**5053. Bogdan, R. C., and Pelecudi, C., Synthesis of four-bar mechanisms on the basis of harmonic analysis** (in Roumanian), *Studii Si Cercetari Mecan. Appl.* **10,** 1, 141–149, 1959.

Synthesis of four-bar mechanisms is treated with the aid of Fourier series set under complex form.

Authors express the kinematic parameter as a function of the rotation angle of the crank which drives the system and study the motion of a point of the connecting rod. There results a parametric equation with the help of which the motion reproduced by the mechanism with respect to the law of variation imposed on one of the parameters is determined. Conditions are given for satisfying the motion laws required at a maximum number of points and also the possibility is established for these laws to be approximated on a working portion of the mechanism with a minimum deviation.

Method is general and shows the usefulness of harmonic analysis for characterizing a mechanism.

N. Tipei, Roumania

**5054. Oderfeld, J., On a certain application of the adjustment calculus to the kinematics of mechanisms** (in Polish), *Zastosowania Matematyki* **4,** 2, 176–194, 1958.

Paper concerns the kinematical analysis of mechanisms, or the determination of velocities and accelerations of every point at any moment. In common practical cases, if the mechanism is complex the analytic method is too tedious and the graphical method not sufficiently accurate. Author proposes a new method consisting of a few stages.

In stage (a), by the usual graphical method, the positions of the mechanism are determined at moments equidistant in time. The results obtained contain relatively large accidental errors.

In stage (b) these errors are smoothed by using the method of the movable mean value. Such a method has not yet been used for this purpose.

In stage (c) the velocities and the accelerations are determined by means of new interpolation formulas enabling direct calculation from non-corrected data obtained in stage (a).

In stage (d) the correctness of the calculations is verified, and in (e) the errors are analyzed in detail, enabling the determination of the errors of the mean unsmoothed and smoothed positions and those of the mean velocities and accelerations, on the basis of the differences between the smoothed and the unsmoothed positions; the differences may be calculated in every particular case.

The method proposed is illustrated by the example of a locomotive valve gear and by a somewhat artificial example in which the sequence of positions is determined by a stochastic process. This makes possible a comparison of the results by the above method with the accurate solution. Cz. Rajski, Poland

**5055. Popov, A. F., Bases of the theory of the contour construction of kinematic chains and their applications for the determination of the degree of mobility** (in Russian), *Nauch. Zap. L'vovsk. Politekhn. In-ta* no. 43, 158–166, 1956; *Ref. Zh. Mekh.* no. 4, 1958, Rev. 3745.

The existing procedure for the structural analysis of complex mechanisms of different classes is developed. Means are indicated for determining to which class the mechanisms being investigated belong. V. A. Zinov'ev

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5056. Kurganov, M. M., The kinematics and dynamics of an automatic coupling mechanism** (in Russian), *Trudi Mosk. In-ta Inzh. Zh.-d. Transp.* no. 82/3, 175–185, 1955; *Ref. Zh. Mekh.* no. 11, 1957, Rev. 12461.

Results are published of the theoretical investigations of the work of the mechanism of an automatic coupling at the time of coupling and when the train is in motion. Recommendations are made for raising the reliability factor of the automatic coupling mechanism. A. A. D'chko

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5057. Thurgood, D. A., Interference fits—a method of solving the equation for design office use,** *Aircr. Engng.* **31,** 359, 6–7, Jan. 1959.

This article is a collection of stress equations and some suggested criteria for designing interference fits. Two-dimensional stress equations are taken from elasticity theory for two thick-walled cylinders, one shrunk onto the other, with an internal

pressure in the smaller cylinder (bush). Operation of the assembly at temperatures other than room temperature is also considered. Design criteria include excessive stresses or deflections in either the bush or the housing, buckling of the bush, or separation of the shrunk-fit surfaces in service by differential thermal expansion of the parts. Elasticity and thermal expansivity properties for some typical engineering materials are listed. The design of a phosphor bronze bush shrunk into an aluminum housing is outlined, including some graphs.

Reviewer believes that article would be useful in designing most shrink-fit bushes. However, axial stresses and edge effects, which are neglected, may be important in highly-stressed interference fits, particularly with brittle materials. The derivations appear correct. Reviewer found two minor errors only: In Eq. [1] for the interference, the sign of the first term after the end of the square brackets should be minus instead of plus, and the factor $E_0$ should be inserted in the last term.

L. B. Sibley, USA

**5058. Herrington, M. G., Recent developments and modern techniques in the use of gaskets,** *Proc. Instn. Mech. Engrs., Auto. Div.* no. 5, 183–186, 1956–57.

**5059. Hepworth, J. L., Sealing rings for wet cylinder liners,** *Proc. Instn. Mech. Engrs., Auto Div.* no. 5, 175–182, 1956–57.

**5060. Cross, R. C., Gudgeon-pin location,** *Proc. Instn. Mech. Engrs., Auto. Div.* no. 5, 193–196, 1956–57.

**5061. Gerts, E. V., Certain questions of the structure and classification of pneumatic apparatus** (in Russian), *Trudi Inst. Mashinoved., Akad. Nauk SSSR* **16,** 63, 5–16, 1956. (Translation by Morris D. Friedman, Inc., P. O. Box 35, W. Newton, Mass., G-143, 13 pp.)

## Fastening and Joining Methods

(See Rev. 5188)

## Rheology

**5062. Noll, W., A mathematical theory of the mechanical behavior of continuous media,** *Arch. Rational Mech. Anal.* **2,** 3, 197–226, Dec. 1958.

Author attempts to clarify formulations of continuum mechanics, concentrating on concepts and principles relevant to the formulation of constitutive equations. He presents a general framework which includes most if not all existing theories of continuum mechanics. He gives a new and rather appealing scheme for treating material symmetry. Reviewer feels that there may be materials which are neither fluids nor solids as author defines these. He does not make any claim to the contrary.

This work, together with the researches of Green and Rivlin [*Arch. Rational Mech. Anal.* **1**, 1, 1–34, 1958] and Green, Rivlin and Spencer, [*Arch. Rational Mech. Anal.* **1**, 1, 82–90, 1959] promise to provide a basis for constructing adequate nonlinear viscoelasticity theories. These investigations make effective use of functional analysis, an area of mathematics which has been sadly neglected by workers in mechanics.

J. L. Ericksen, USA

**5063. Criminale, W. O., Jr., Ericksen, J. L., and Filbey, G. L., Jr., Steady shear flow of non-Newtonian fluids** (in English), *Arch. Rational Mech. Anal.* **1**, 5, 410–417, Aug. 1958.

The non-Newtonian fluids considered here are assumed to obey a constitutive equation of the differential type as proposed by Rivlin and Ericksen [*J. Rational Mech. Anal.* **4**, 323–425, **1955**; AMR **6** (1955), Rev. 3656]. Authors study the possibility of steady rectilinear flow through pipes of arbitrary cross section. They show that such flow is impossible for noncircular cross sections and hence that secondary flows must occur unless the material functions characterizing the fluid satisfy certain relations. The experimental evidence in favor or against the validity of these relations is discussed. A partial proof is given to show that if secondary flow does not occur in elliptical tubes it will not occur in tubes of any other shape.

W. Noll, USA

**5064. Theodorides, P., Parallel effects of bulk viscosity and time lag in kinetics of non-monatomic fluids** (in English), *ZAMP* **9b**, 5/6, 668–686, 1958.

Author considers the bulk viscosity and time lag in the kinetics of non-monatomic fluids involving one inner process such as a single mode of vibration within molecules. For a triaxial flow with trimerous viscosity, equations of motion, continuity and energy have been derived, and effects of bulk viscosity and quadratic shear viscosity on the Navier-Stokes equations are discussed. Utilizing the data on bulk viscosity for molecular nitrogen from ultrasonic measurements, the end state characteristics of shock waves have been calculated.

G. S. Verma, USA

**5065. Paslay, P. R., and Slibar, A., Criterion for flow of a Bingham plastic between two cylinders loaded by torque and pressure gradient,** *J. Appl. Mech.* **25,** 2, 284–285 (Brief Notes), June 1958.

## Hydraulics

(See also Revs. 4826, 4999, 5087, 5320)

**Book—5066. Valembois, J., Notes on applied hydraulics [Memento d' hydraulique pratique],** Paris, Eyrolles, 1959, 108 pp. 950 Fr.

This book, in a clear and concise way, is especially intended to give to students in engineering the essential knowledge for the solution of problems in industrial hydraulics—mentioning at the same time the basic principles of theoretical fluid mechanics. The different questions are summarily treated, showing the basic hypothesis, the methods of mathematical or graphic solution and the main theoretical or experimental results obtained. Many examples aim at indicating scales of figures, units, etc.

This work contains 15 chapters, solving most of the problems occurring in industry. At the end of every chapter author gives a bibliography. These bibliographies include the most modern basic books by outstanding specialists.

In a supplement, two chapters (for students in engineering only) contain: first, a series of plain questions by which students may test their understanding of the knowledge summarized in the book; second, exercises, the solution of which lead to an exact notion of the quantitative scales.

This work is of high interest to nonspecialist engineers. Without any tedious research, they will find in it all the elements of basic hydraulics, together with the addresses of the centers of studies and the specialized laboratories.

L. Escande, France

**5067. Kartvelishvili, N. A., Energetic multiannual regulation of river flow in some complex cases** (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 6, 102–109, June 1957.

Author derived and solved a general integral equation (in Riemann integrals) for frequency distribution of storage volume.

Only annual runoff volume and annual component of storage were considered, seasonal change in runoff and seasonal storage component were neglected. Annual use of water was assumed proportional to annual component of storage at end of preceding year. General equation is as valid for annual runoff, which is random in occurrence, as for inflow dependent on antecedent annual runoff. Second problem solved by author was derivation and solution of integral equations for the frequency distribution of energy production by two hydroelectric stations, working in parallel, with two storage reservoirs for annual regulation on two different rivers. In this case a stochastic relationship between annual runoff for each river was assumed, but annual runoff of each river was considered as independent of runoff of preceding year.

Both problems were solved with the use of relationship between conditional probability of an event and unconditional probability distribution of annual runoff.

For solution of the second problem author indicated a method for derivation of equation of combined probability distribution for annual runoff of two rivers with some stochastic relationship between their runoff, and developed some auxiliary formulas in introductory part. Author explained physical significance and limitations of mathematical solution of given equations.

L. J. Moskvitinoff, USA

**5068. Walker, W. J., The continuous and accurate proportioning of liquids under variable flow conditions with particular reference to a new method,** *Trans. Instn. Chem. Engrs.* **37**, 1, 9–13, Feb. 1959.

Paper describes a flotation tank method which was first tried as a means of dosing the drinking water of cattle with phosphate solution. The theory of this is given. Tests showed, as expected, a variation in the dosage ratio, which was, however, within allowable limits. At the same time, it appeared that this variation might well be corrected by suitable spring control of the flotation movements. Analysis, which comprises the bulk of the paper, shows that this is possible. The system can be designed for any pressure conditions and provides a constant ratio of delivery for any number of liquids under all changes of pressure and velocity conditions.

From author's summary

**5069. Sutherland, C. D., and Cohen, H., Finite cavity cascade flow,** Proc. Third U. S. Nat. Congr. Appl. Mech., June 1958; Amer. Soc. Mech. Engrs., 1958, 837–846.

Steady flow of an incompressible, nonviscous fluid past an infinite array of lifting foils under such conditions that fully developed cavities appear on the foil surfaces is considered. The foils are taken to be thin and at small angle of attack so that the linearized theory of Tulin may be applied. The linearized problem is solved for any stagger angle and any chord-foil spacing ratio in terms of the cavitation number, cavity length, exit angle, and exit velocity. The effect of changing chord-foil spacing ratio on cavitation number, exit angle and lift coefficient is obtained as well as the relation between lift coefficient and cavitation number. Limiting cases of the solution, the isolated foil and the cavity of infinite length, agree with previous results.

From authors' summary

**5070. Haskind, M. D., Oscillation of a cascade of thin sections in incompressible flow,** *J. Appl. Math. Mech. (Prikl. Mat. Mekh.)* **22**, 2, 349–354, 1958. (Reprint order no. PMM 30, Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Author determines circulation I for the stationary cascade and modifies this result to account for the nonstationary effects during oscillation of the cascade. The magnitude of the hydrodynamic force existing on a blade of the cascade is found and this force is separated to show the relation of the effect of the apparent mass.

Author assumes (1) the normal velocity $v_n$ and (2) the velocity potential vary exponentially with time. The complex potential is separated into two functions $w_1(z)$ and $w_0(z)$, where $w_0(z)$ defines the noncirculatory flow and $w_1(z)$ is the solution of the homogeneous problem. Conditions to be satisfied are imposed and conformal mapping is used to find two functions $F_1$ and $F_2$ which represent a velocity potential function leading to the distribution of sources and the effect of a distribution of vortices along the segments. Cascade theory is applied and the circulation is determined.

W. H. Sellers, USA

**5071. Hincu, S., Determination of the stabilizing length for the motion of fluids with free surface in large channels with horizontal bottom** (in Roumanian), *Studii Si Cercetari Mecan. Appl.* **10**, 1, 109–116, 1959.

Paper discusses the stabilizing length for the laminar and turbulent motion of fluids with free surface, in large channels with horizontal bottom.

Author proceeds from the boundary-layer integral equations and obtains results which express a functional dependence between the relative stabilizing length and the geometric and hydraulic characteristics.

D. Gh. Ionescu, Roumania

**5072. Samoilovich, G. S., and Sherstyuk, A. N., Analysis of curvilinear axisymmetric channels** (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 4, 78–81, Apr. 1958.

An approximate method for calculating incompressible potential flow in curvilinear axisymmetric channels is presented under the following simplifying assumptions:

(1) The equipotentials are represented by circular arcs.

(2) The velocity $c$ on the equipotential is assumed to have the form of

$$c(r_a + b + kb^2) = \text{const}$$

where $r_a$ is the radius of curvature of the inner wall, $b$ is the distance on the equipotential from the inner wall, $k$ is a constant for the given equipotential.

(3) The distance from the axis of symmetry on the equipotential is approximated by a linear function of $b$.

Two numerical examples calculated by the above method show a good agreement between the results of this approximate method and those of an exact solution. However, the method is not suitable for calculating the inlet and outlet sections of the curvilinear axisymmetric channels.

J. Polasek, Czechoslovakia

**5073. Iwasa, Y., Theoretical study of hydraulic behaviors of boundary characteristics on channel transitions and controls in divergent channel** (in Japanese), *Trans. Japan Soc. Civ. Engrs.* **59** (Separate 3-1), 1–32, Nov. 1958.

As one of the research projects of clear formulation in hydraulic characteristics of transitional behaviors from subcritical to supercritical or vice versa, this paper deals with the hydraulic behaviors of boundary characteristics in shape, roughness and bed slope on channel transitions and controls in divergent or convergent channel.

The open channel flow in divergent or convergent channel is changed in its flow regimes by the existence of singular points of water surface profile equation based on the one-dimensional procedure in gradually varied flows, and therefore the theoretical approach in analysis of transitional flows will be established by the application of the theory of singular point in ordinary differential equation and the topological method in nonlinear mechanics.

The classification of singular points and hydraulic significance of transitional characteristics of flows passing through the singular point and, especially from a standpoint of practical application, those for the Chézy and Manning flows are presented.

The present procedure of analysis is applied not only to the flow in divergent or convergent channel but also to the flow with side in- or outflow and similar problems in hydraulics of open channel flows.

Finally, the application of the analysis described here to the discharge measurement of open channel flows is briefly discussed.

From author's summary

*Courtesy, Editorial Committee of the Japan Society of Civil Engineers*

**5074. Gilyarov, N. P., Some outstanding questions in the utilization of aerodynamic models of rivers** (in Russian), *Trudi Leningrad In-ta Inzh. Vod. Transp.* no. 24, 31–41, 1957; *Ref. Zh. Mekh.* no. 4, 1958, Rev. 4143.

In order to prevent the distortion of the outline of the distribution of velocities along the vertical, introduced by friction of the air flow on aerodynamical models against the glass, depicting the free flow of the flow, author proposes to lift the glass on the model to a height of $\Delta H$, at which the maximum velocities on the vertical in nature and in the model would have identical values and an identical height position relative to the bottom. In this way the proposed method of modelling is founded on the acceptance as the modelling part of only part of the air flow by the depth —from the bottom up to the (level) maximum velocities on the vertical. The upper part of the flow below the glass participates in the work of the model, but is not taken into account in the calculations. For the determination of the relative coordinate $\eta$ of the maximum value for the velocity on the outline, author introduces a graph for the value $\eta$ as a function of the relationship of the coefficient of roughness of the glass and the model's bottom. Formulas are derived for the conditions of kinematic similarity of the planned flow on the model according toV. M. Makkaveev and scale correlations for slopes, depths and velocities. At the end of his article author puts forward his views on the work of A. G. Averkiev [*Izd. Vses. N.-I. In-ta Gidrotekhn.* **52**, 1954] containing criticisms of the author's method, embodying a detailed analysis of the critical position taken by A. G. Averkiev.

A. M. Latyshenkov

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5075. Modebadze, Z. K., Hydraulic ram system for energy accumulation in small hydroelectric stations** (in Russian), *Trudi In-ta Energ. Akad. Nauk GruzSSR* **10**, 19–29, 1956; *Ref. Zh. Mekh.* no. 4, 1958, Rev. 4139.

Two schemes are examined for the accumulation of energy in small hydroelectric stations during seasonal control, based on the utilization of hydroram feeders (hydraulic rams). Formulas are derived which enable a determination of the economic parameters for HES and HAV, fitted with hydraulic rams, to be made. An example of the calculation is given. V. I. Vasil'ev

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5076. Gigiberiya, G. Ya., Hydraulic shock when air inclusions issue from the pressure pipe system** (in Russian), *Trudi In-ta Energ. Akad. Nauk GruzSSR* **10**, 195–203, 1956; *Ref. Zh. Mekh.* no. 4, 1958, Rev. 4100.

Taking as the basis for his paper the author's previous work [*Trudi In-ta Energ. Akad. Nauk GruzSSR* **10**, 67–68, 1956] a method is indicated for the determination of the principle of change of water discharge in the section of a water pipe adjacent to a reservoir of sufficiently large dimensions, at the time when air inclusions are issuing from the water pipe. This principle appears to be the boundary condition for the calculation of hydraulic shock, which can be carried out by means of any one of the known methods. In the calculation furnished by the author, the known method of M. A. Mostkov is used.

N. A. Kartvelishvili

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5077. Ivanova, A. I., Screw motion of a viscous incompressible liquid** (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 12, 46–50, Dec. 1957.

The problem of slow propagation of a viscous substance along a tube by the action of a rotating helicoidal surface (screw) is first formulated by the aid of screw-coordinates and then solved in terms of particular harmonics related to Bessel functions of imaginary index. Mathematical difficulties, however, preclude the application of this solution; so the assumption of large screw thread is introduced which permits the working out of the solution up to the final result, giving the rate of flow of the substance as linear function of the axial speed of the screw and the pressure gradient.

The article is difficult to read because of its mathematical conciseness. Russian references are given, however, and could possibly be helpful. A. Von Baranoff, France

**5078. Baines, W. D., Laminar flow of dilute fibre suspensions,** *Nat. Res. Counc. Canada, Div. Mech. Engng. Rep.* MH-73, 16 pp., Aug. 1958.

Flow of dilute fiber suspensions such as paper stock is considered on the basis of observed characteristics. It is assumed that there is a separation into two regions, one moving as a solid body and containing all of the fiber, and the other a thin film of liquid. Flow in pipes is considered in detail, particularly the consequences of this type of motion on the friction characteristics. Flow between rotating concentric cylinders is also considered, and it is shown how this type may be used to predict the friction properties of pipes.

From author's summary by P. Franke, Germany

## Incompressible Flow

(See also Revs. 5063, 5109, 5116, 5199)

**5079. Andersson, B. J., On two-dimensional vortex motion in a simply connected region** (in English), *Trans. Roy. Inst. Technol. Stockholm* no. 126, 6 pp., 1958.

The path of a vortex inside a simply connected region bounded by a rigid wall is found in terms of a mapping function which maps the region onto a unit circle with the vortex at its center.

A. F. Pillow, Australia

**5080. Sarpkaya, T., Oblique impact of a bounded stream on a plane lamina,** *J. Franklin Inst.* **267**, 3, 229–242, Mar. 1959.

The Schwarz-Christoffel transformation is used to solve the problem of free-streamline flow past a plane lamina between two parallel walls. Experimental results supporting the theoretical predictions for the coefficient of discharge are also presented.

C.-S. Yih, USA

**5081. Mimura, Y., Phenomenological free streamline theory. Part I. Flow past a normal plate,** *J. Phys. Soc. Japan* **13**, 11, 1361–1374, Nov. 1958.

Author considers the following problem: Given a pressure distribution on the initial portion of the free streamline, either from assumption or experiment, how accurately can the pressure distribution on the front of the plate and the shape of the free streamline be determined? This is of interest, since such problems are usually solved by making an assumption about the free streamline (in the hodograph plane). Author sets up equations, based on Woods' method, and applies them to several theoretical models of flow past a flat plate, as well as to experimental cases. Results indicate that the pressure distribution on the front of the plate depends mainly on the initial portion of the free streamline, but the shape

of the free streamline is more sensitive. Effects due to changes on the free streamline for downstream are evaluated.

A. Roshko, USA

**5082. Russell, T. W. F., and Charles, M. E., The effect of the less viscous liquid in the laminar flow of two immiscible liquids,** *Canad. J. Chem. Engng.* **37**, 1, 18–24, Feb. 1959.

Paper deals with fully developed laminar, horizontal flow of two immiscible Newtonian liquids, stratified into (1) two layers between wide parallel plates and (2) a concentric layer in a circular tube. Expressions were deduced for the volumetric flow rate in terms of liquid viscosities, pressure gradient and conduit geometry. Position of interface for minimum pressure gradient and power requirement were examined and discussed. Computed results for the variation of flow rate per unit pressure gradient with interface location were presented graphically for range of viscosities from 1 to 1000 c.p. for one liquid, the other being water with $\mu = 1$ c.p. When the predicted pressure gradients were compared with the experimental data of Clark, Russell, et al, the observed values fell between those calculated for concentric and stratified flow in a circular tube.

B. T. Chao, USA

**5083. Yen, K. T., A study of the use of optimal coordinates for some viscous flow problems,** AFOSR TN 59-11 (Rensselaer Polytech. Inst., Dept. Aero. Engng. TR AE 5805; ASTIA AD 208 592), 43 pp., Apr. 1958.

A study of the use of optimal coordinates for (a) boundary-layer flows of the wedge type and (b) a simple shear flow over a flat plate is made. It is shown that, for these problems, optimal coordinates can be found to achieve smooth transition from the inner viscous flow to the "outer" free-stream flow. The significance of these coordinates and some features of the physical problems are discussed.

In particular, the boundary-layer problem of a flat plate is considered in some detail: a new definition of boundary-layer thickness is proposed and some points concerning the properties of the Blasius solution and the boundary-layer solution in parabolic coordinates are discussed. For the simple shear flow it is shown that parabolic coordinates can be used as the optimal coordinates. Two methods of solution for this problem with vorticity index $\omega_0 \sim 1$ and $\omega_0 \ll 1$, respectively, are given.

From author's summary

**5084. Filippova, L. A., Transient motion of a viscous incompressible liquid in a narrow slot of constant width** (in Russian), *Ucb. Zap. LGU* no. 217, 225–235, 1957; *Ref. Zb. Mekb.* no. 4, 1958, Rev. 4207.

The problem is examined of the transient turbulent motion of an incompressible liquid in a narrow slot, the lower wall of which moves in accordance to the given principle, in the presence of a fall of pressure, appearing as a known function of time. The flow is assumed to be approximately turbulent over the whole section of the slot, so that at the wall the liquid has a velocity differing from zero, the velocity of slipping past. For the determination of the values of the velocities of slipping through and of the relations between the coefficients of turbulent exchange and viscosity, data for the semiempirical theories are used. It is assumed that the values of these magnitudes as obtained for a steady flow also hold good for a transient flow. As the result of integration of the motion, formulas are obtained for the determination of the relation of the velocities outline to the discharge by time.

B. N. Rumyantsev

*Courtesy Referativnyi Zhurnal, USSR*

*Translation, courtesy Ministry of Supply, England*

**5085. Tretiakov, M. V., Flow around permeable contours,** *Appl. Math. Mech. (Prikl. Math. Mekb.)* **22**, 2, 297–304, 1958. (Reprint order no. PMM 24, Pergamon Press, 122 E. 55th St., New York 22, N.Y.)

Mathematical treatment of the general problem of flow around permeable contours is presented in detail. To solve the problem it is assumed that: (1) permeable contours are lines of discontinuity in velocity and pressure; (2) the flow is steady and irrotational; and (3) the contours are uniformly permeable. An example of the solution is given for the case of flow of a potential stream around a uniformly permeable circle.

S. R. Faris, USA

**5086. Dumitrescu, D., and Ionescu, D. Gh., Numerical methods for studying axisymmetric motions of ideal fluids** (in Roumanian), *Studii Si Cercetari Mecan. Appl.* **9**, 4, 919–935, 1958.

Determination of the characteristics of the axisymmetric flow of an incompressible ideal fluid through a circular conduit in which a body is placed. Within a first calculation method, authors proceed from the expression of velocity potential and from that of the stream function set under the form of Bessel functions of the first kind and of the first- and second-order, respectively. A procedure for calculating the coefficients of these series is given with application to the case in which the conduit contains a diaphragm or an axisymmetric body. The coefficients are the solutions of a system of $n$ linear algebraic equations with $n$ unknowns. As a general proof of the convergence cannot be given, authors suggest the study of convergence for each particular case. The second calculus uses the relaxation method which determines only the stream lines. Numerical computations are similarly performed in case of a diaphragm, of a sphere and of an ellipsoid.

T. Oroveanu, Roumania

**5087. Moklyak, V. I., Graphical flow curves for a rectangular waterway** (in Russian), *Dop. Akad. Nauk URSR* no. 6, 527–531, 1956; *Ref. Zb. Mekb.* no. 4, 1958, Rev. 4124.

Author obtains, by analytical methods, a graphical flow-curve for a rectangular channel. Starting from the stepped dependence of change of velocity of the flow along the vertical and along the width of the channel, author records an equation for the flow discharge in a symmetrical channel

$$Q = \int_0^{\omega = f(v)} v \, d\omega =$$

$$= 2v_{max} \int_0^x \int_0^{y = f(x)} \left(1 - \frac{y}{b}\right)^{\frac{1}{m}} \left(\frac{b}{H}\right)^{\frac{1}{n}} \left(1 - \frac{2x}{b}\right)^{\frac{1}{p}} dx\, dy \quad [1]$$

Here $b$ is the depth of the flow on a given vertical; $H$ the maximum depth of the flow on the mean vertical; $x$ the distance from the mean vertical to the vertical with the depth $b$; $v_{max}$ the maximum velocity of the flow at the surface at the mean vertical; $b$ the width of the channel at depth $y$; $m$, $n$, $p$ are constants. Integral [1] is only taken in its general form for the case of a rectangular channel. Adopting for such a case $b = H$ and $b = B$ and a velocity distribution

$$v = v_{max} \left(1 - \frac{y}{H}\right)^{\frac{1}{m}} \left(1 - \frac{2x}{B}\right)^{\frac{1}{p}} \quad [2]$$

author obtains an expression for the discharge within the limits of change $\beta = v/v_{max}$ from 0 to $\beta$

$$Q = \frac{m}{m+1} \frac{p}{p+1} v_{max} BH \frac{(p+1)\beta^{m+1} - (m+1)\beta^{p+1}}{p - m} \quad [3]$$

or in the relative values $\alpha = Q/Q_{max}$ and $\beta$.

$$\alpha = \frac{(p+1)\beta^{m+1} - (m+1)\beta^{p+1}}{p - m} \quad [5]$$

For the case of the logarithmic principle of change of velocity

along the vertical, author investigates the equation of the discharge graphic curve and obtains an expression for the total yield

$$Q = \frac{mpBHv_{max}}{(m+1)(p+1)} \qquad [6]$$

Making use of the data available in the literature, author obtains relationships for the parameters $m$ and $p$, connecting them with the Prandtl-Karman and Chezy coefficients. A sufficiently satisfactory convergence of the relationships obtained with the experimental data by Bazen is established for a wooden rectangular trough. However, their practical application at the present time is not possible, as data regarding the value of the Prandtl-Karman coefficient for concrete cases in engineering practice is not available.

A. A. Uginchus

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5088. Abramovich, G. N., Flow of air in the presence of regions of reverse currents** (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 12, 7–14, Dec. 1957.

By applying an intuitive analysis to the complex problem of a separated flow region behind a "step" in the wall of a channel, author succeeds in breaking it up into three partial problems which admit treatment by known approximate methods. These partial problems are: turbulent mixing of two opposed jets in the region immediately behind the step; reversal of the flow by 180° in the rearward section of the circulatory region (here the assumption of ideal flow is made); gradual adjustment of the flow behind the separated region.

Reviewer finds this discussion rather stimulating. It certainly constitutes a starting point from which refinements could be tried.

A. Von Baranoff, France

**5089. Odel'skii, E. Kh., Equation for the movement of low pressure gas in pipe lines** (in Russian), *Sb. Naucb. Trudí Belorussk. Politekbn. In-ta* no. 56, 143–152, 1957; *Ref. Zb. Mekb.* no. 4, 1958, Rev. 3962.

An empirical formula is put forward for the determination of pressure loss in low-pressure gas during its passage through pipelines

$$\Delta p = 17 \times 10^5 \frac{V^{1.75}}{D^{4.75}} \gamma l$$

where $\Delta p$ is the fall of pressure in mm (water pressure); $V$ the hourly consumption of gas in $m^3$; $\gamma$ the specific weight of the gas in $Kg/m^3$; $D$ the diameter of the pipeline in mm; $l$ the length of the pipeline in m. Tables have been prepared for the determination of pressure loss due to friction, worked out per running meter of length when $\gamma = 1$. An example of the calculations is given.

V. D. Sokolov

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5090. Smyslov, V. V., Calculation for the through-put capacity of a pipe line with gas flowing through at great speeds** (in Russian), New items in constructional technics no. 9, Kiev, 1956, 137–142; *Ref. Zb. Mekb.* no. 4, 1958, Rev. 3964.

Based on the equations of the motion of gas in a cylindrical tube, formulas are obtained for the calculation for the through-put capacity of a long pipeline at its outlet to the air. Author shows that the calculation for the through-put capacity of a pipeline, when computed by the earlier accepted formula for the critical discharge from an orifice, even when introducing the coefficient $\varphi$, which takes into account the losses along the pipeline, gives results markedly differing from the precise values.

I. S. Simonov

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5091. Prausnitz, J. M., Regular solution theory for gas-liquid solutions,** *AIChE J.* **4,** 3, 269–272, Sept. 1958.

Regular-solution equations developed by Hildebrand and others are useful for obtaining semiquantitative estimates of phase equilibria in nonpolar systems. These equations, however, do not take into account the volume change on mixing and therefore require modification for solutions of gases in liquids. The required modifications are presented in this paper, and the resulting equations give reasonable estimates of the solubilities of gases and of the temperature coefficient of solubility. In the modified form the regular-solution theory may be used to estimate gas-liquid phase equilibria at high pressures.

From author's summary

## Compressible Flow (Continuum and Noncontinuum Flow)

(See also Revs. 4824, 4867, 5112, 5113, 5119, 5120, 5121, 5139, 5150, 5162, 5163, 5164, 5165, 5211, 5256)

**5092. Mackie, A. G., The calculation of the drag in problems solved by the hodograph method,** *Phil. Mag.* (8) **3,** 26, 140–142, Feb. 1958.

Explicit formulas are found for the total drag of such obstacles whose profiles consist of segments along which either the magnitude or the direction of velocity vector is constant. The object of this note is to show that the drag on a straight segment of a body can be found directly from the solution in terms of the velocity variable. Formulas are given for incompressible flow and for the exact equation of compressible flow.

F. W. Keune, Germany

**5093. Krajewski, B., Application of the variational method to the problem of axisymmetrical rotational compressible flow** (in English), *Arch. Mech. Stos.* **9,** 2, 211–216, 1957.

Equations for enthalpy and continuity are reduced to one equation with the usual assumptions. This equation is then transformed in such a way that the thermodynamical values are expressed as functions of a stream function and the radial coordinate. It is shown that this equation satisfies a variational condition for the stream function.

J. H. van de Stadt, Holland

**5094. Ryzov, O. S., Some degenerate transonic flows,** *Appl. Math. Mech. (Prikl. Math. Mekh.)* **22,** 2, 355–361, 1958. (Reprint order no. PMM 31, Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Starting from the transonic approximations for three-dimensional gas flow in Cartesian coordinates, author seeks double waves in which the perturbation velocity $u$ in the direction of the undisturbed velocity vector (the $x$-axis) may be expressed as a function $u = u(v,w)$ of the perturbation velocity components along the other two axes. The component $u$ satisfies a nonlinear equation expressible in terms of the hodograph variables $v,w$ alone. The Legendre potential satisfies a similar equation with identical coefficients of the highest derivatives. Solutions are found by use of transfomation groups; they generalize various well-known particular solutions. By means of a limiting process developed in a paper by Barenblatt [*Prikl. Mat. Mekh.* **18,** 1954] a further class of solutions is derived, but no indication is given of its physical significance.

D. C. Pack, Scotland

**5095. Kopylov, G. N., Aerodynamic characteristics of thin wedge-shaped profiles in transonic flows,** *Appl. Math. Mech. (Prikl. Math. Mekh.)* **22,** 1, 182–191, 1958. (Reprint order no. PMM 13, Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Paper investigates, within the transonic approximation for a Tricomi gas, by the aid of hodograph methods, the flow past a thin double-wedge profile at a small angle of attack in a sonic stream. The author continues his solution for the subsonic domain [*Prikl. Mat. Mekh.* **21**, no. 1, 1957] into the supersonic region, downstream of the limiting characteristics, on the basis of three simplified models for the flow in this region. These are (1) flow analogous to that past a wedge at zero angle of attack with stagnation point at the nose, (2) flow over upper and lower front faces of the profile analogous to that over upper and lower faces of a flat plate at an angle of attack with stagnation point on the lower face, (3) flow over the whole profile analogous to that around a flat plate at an angle of attack. The boundary-value problems are solved by means of a mapping onto the characteristic plane, and the solution is given in terms of an integral operator of the solution in the subsonic domain.

The pressure distribution is computed and a reasonable degree of quantitative agreement is found between model (1) and the results of Guderley and Yoshihara [AMR **5** (1952), Rev. 3149]. Lift, drag and moment coefficients are obtained for model (1). Estimates are made of the bounds of velocity of model (2). The paper is highly condensed and the difficulty of its reading is increased by a number of typographical errors (which were present in the original Russian paper). J. B. Helliwell, Scotland

**5096. Landahl, M. T., Theoretical studies of unsteady transonic flow. Part II. The oscillating semi-infinite rectangular wing** (in English), *Flygtekhn. Försöksanst. Medd.* 78, 20 pp., 1958.

Based on the linearized equation for transonic flow, the aerodynamic forces on an oscillating semi-infinite rectangular wing are calculated by use of Fourier transforms. The results are given in terms of end corrections of two-dimensional strip theory.

Applying the results, an approximation of forces on a finite rectangular wing for large values of $Ak^{1/2}$ ($A$ aspect ratio, $k$ reduced frequency) is given by assuming the mutual side-edge interference may be neglected. [For Part I, see AMR **12** (1959), Rev. 2466.]

L. N. Tao, USA

**5097. Landahl, M. T., Theoretical studies of unsteady transonic flow. Part III. The oscillating low-aspect-ratio rectangular wing** (in English), *Flygtekn. Försöksanst. Medd.* 79, 15 pp., 1958.

Part III extends author's studies to an approximate finding of aerodynamic forces on an oscillating rectangular wing of low aspect ratio. The solution in an asymptotic series is valid for small values of $k\sigma^3$ ($\sigma$ semispan-to-chord ratio, $k$ reduced frequency). The result is compared with the slender-wing theory. Near the upper limit present results are in agreement with those of large aspect ratio of Part II [see preceding review].

L. N. Tao, USA

**5098. Carafoli, E., and Sandulescu, S., Aerodynamic characteristics of harmonically oscillating ailerons in supersonic flow** (in Roumanian), *Studii Si Cercetari Mecan. Appl.* **10**, 1, 13–40, 1959.

Paper concerns the harmonic oscillations of ailerons about their hinge, assuming a supersonic regime and a low frequency of oscillations. The reduced potential is used and the case of polygonally shaped ailerons is discussed for which the oscillating motion is considered as originating from two other harmonic oscillations of the same frequency and having the same phase at the initial moment. Authors show that the problem may be reduced to the study of two homogeneous motions of the first and second order. The polygonally shaped aileron with subsonic leading edge is first treated and the lift and drag coefficients are determined, the intervening suction force being included in the latter coefficient. The rolling, pitching and hinge moment coefficients are also calculated. The same characteristics are determined for the polygonally shaped aileron with supersonic leading edge. Results are then applied to several particular cases, namely, triangular, trapezoidal and rectangular ailerons.

T. Oroveanu, Roumania

**5099. Volkova, L. P., and Iudelovich, M. Ia., Shock losses in stepped tubes for supersonic pressure ratios** (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 4, 67–72, Apr. 1958.

A theoretical and experimental investigation into pressure recovery in a jet issuing from a supersonic nozzle (throat section $F_*$, outlet section $F_b$) into a cylindrical pipe with cross section $F_H > F_b$.

Theoretically, it is assumed that the jet continues to expand supersonically to a cross section $F_1 (F_b < F_1 < F_H)$, then, after a normal shock, expands subsonically-in fact incompressibly.

The ratio $P_0/P_H$, of initial total head to static outlet pressure, together with $F_H/F_*$, determine $F_1$. Provided $P_0/P_H$ does not exceed the value making $F_1 F_H$ the experimental values confirm the theory.

Reviewer believes the results to be sound, and in no way surprising. The presentation is marred by undefined symbols.

A. H. Armstrong, England

**5100. Pantazopol, D., Approximate determination of pressure distribution on a body placed in supersonic gas flow** (in Russian), *Acad. Republ. Pop. Romine, Rev. Mecan. Appl.* **3**, 3, 234–251, 1958.

The plane gas flow is considered in a subsonic region bounded by the surface of a body, a detached shock wave in front of it and the sonic lines, under the approximate assumption that the streamlines do not differ sensibly from those of a suitably chosen incompressible flow. This flow is then determined using a hodograph (Helmholtz) transform and a conformal mapping on a half-annular domain. The pressure distribution along the body surface is then computed by applying one-dimensional compressible flow rules in the tube formed between the impact line, prolonged by the body line and an indefinitely close streamline. Due to the fact that, in the vicinity of the impact point and of the body surface, the gas velocity is low, the approximate assumption used yields results which are very close to experimental values. Method is applied to several contour forms of the body. The computed detachment distance of the shock wave also agrees well with experimental data.

L. Dumitrescu, Roumania

**5101. Vaglio-Laurin, R., and Ferri, A., Theoretical investigation of the flow field about blunt-nosed bodies in supersonic flight,** *J. Aero/Space Sci.* **25**, 12, 761–770, Dec. 1958.

Paper presents a numerical method for solving the inverse problem of blunt body flow (i.e. determining flow field and body shape behind a given detached shock). A perturbation scheme is also given for treating the direct problem (i.e. determining flow field and shock shape for a specified body shape) at zero and small angles of attack.

A specific numerical example treats the inverse problem for a given axisymmetric parabolic shock. Unfortunately, no details of the flow characteristics (Mach number, specific heat ratio) are given. No difficulty with instability was apparently encountered in the numerical computations. W. L. Haberman, USA

**5102. Liu, V. C., On the drag of a flat plate at zero incidence in almost-free-molecule flow,** *J. Fluid Mech.* **5**, 3, 481–490, Apr. 1959.

Paper defines "almost-free-molecule flow" as the regime where dimensions of body of interest are of same magnitude as mean free path, and where departures from rarefied gas flow can be accounted for by collisions of molecules headed for surface of body with molecules diffusely reflected from it. Author calculates the effect of these collisions which diminishes the number of molecules striking the surface, and thus the drag coefficient, in contrast to

true free-molecule flow. Calculation yields qualitatively correct transition curve from free molecule regime to slip flow regime in region where $\sqrt{Re}/M$ is between 0.2 and 3. Author's analysis of collision frequency should prove valuable for other kinetic problems in regime under discussion, but reviewer has one reservation regarding the mean velocity of incident molecules in Eq. [3,8]: velocity distribution of molecules striking a surface, though originating from bulk of gas with Maxwellian distribution, is itself not Maxwellian; their mean velocity is greater than the Maxwellian by a factor $3\pi/8$. E. F. Lype, USA

**5103. Takano, A., Skin friction of a flat plate in the molecular flow of non-Maxwellian gas,** *Trans. Japan Soc. Aero. Engrg.* **1,** 1, 9–18, 1958.

Author claims to calculate the aerodynamic forces on a flat plate in a non-Maxwellian free-molecular flow by starting from the modified velocity distribution function which was used by Bell and Schaaf [AMR **7** (1954), Rev. 2979] in calculating these forces for the case of a cylinder. He obtains an expression for the skin friction in terms of the stresses and heat flow near the plate. After numerous assumptions of dubious validity he then reduces this to a form which is actually Mirels' skin friction [AMR **5** (1952), Rev. 2889] multiplied by a factor to make it agree with the Maxwellian free-molecule value in the limit of large Knudsen number.

D. Pridmore-Brown, USA

**5104. Kanwal, R. P., On curved shock waves in three-dimensional gas flows,** *Quart. Appl. Math.* **16,** 4, 361–372, Jan. 1959.

Author derives expressions for the gradients of velocity, pressure, density, entropy, and vorticity behind a general three-dimensional curved shock wave in a uniform flow with a constant $\gamma$. Results are given in Cartesian tensorial notation. It is shown that the vorticity component normal to the shock surface is everywhere zero, and that the only physically significant shock surfaces through which the flow remains irrotational are the conical and plane shocks. Paper contains relatively little new information, although some known results have been rederived in more elegant and rigorous fashion. L. Talbot, USA

**5105. Zierep, J., Vertical shock wave in a curved channel** (in German), *Dtsch. Versuchsanstalt Luftfahrt Rep.* 51, 36 pp., Apr. 1958.

Author discusses the compression waves formed around a bend by steady, locally supersonic, frictionless flow. Equations of gas dynamics, written in curvilinear coordinates implied by the shape of profile and boundary and shock conditions, point out the geometric form of shocks in the neighborhood of bend. Integration shows that the shock form is a Neil's parabola with both branches possible. The shock is normal to the body, generally with infinite curvature. After the pressure jump, the flow accelerates and pressure decreases; results obtained analytically show good agreement with experiments of Ackeret-Feldmann-Rott.

J. N. Aguirre, Argentina

**5106. Bird, G. A., Some methods of evaluating imperfect gas effects in aerodynamic problems,** *Aero. Res. Counc. Lond. Curr. Pap.* 397, 31 pp., 1958.

Steady and unsteady one-dimensional isentropic flow, Prandtl-Meyer expansion around a corner, normal and oblique shock waves are treated. Instead of solving the fundamental equations by inserting the assumed equation of state for perfect gases, real gas values from published tables [for instance, *Nat. Bur. Stands. Rep.* 3991, Apr. 1955] are substituted and solutions are obtained by numerical and graphical methods. Procedure as outlined by author should be useful in cases where no programs for high-speed computing machines are available. However, for the calculation of shock wave profiles, several programs have recently been described which include not only thermal and caloric imperfections (Bethe-Teller relations) but vibration and dissociation relaxation effects as well [see *J. Chem. Phys.* **28,** p. 1193, 1958; AVCO Research Note 86, Aug. 1958]. C. B. Ludwig, USA

## Boundary Layer

(See also Revs. 5085, 5201, 5202, 5203, 5204, 5206)

**5107. Tanaev, A. A., Effect of the force of gravity on the motion in a laminar boundary layer in longitudinal flow of a gas around a plate,** *Soviet Phys.-J. Tech. Phys.* **3,** 4, 805–813, Dec. 1958. (Translation of *Zh. Tekh. Fiz.* **28,** 4, 862–871, Apr. 1958, by Amer. Inst. Phys., New York, N. Y.)

Solutions are given to the laminar boundary-layer equations for the case of a highly nonisothermal, low Mach number flow of a gas over an inclined flat plate. The momentum equation includes the buoyancy forces. The cases of a linear temperature-viscosity relationship and also a power-law relationship are considered.

The final results appear as expressions for $C_f/C_{f0} = /[(Gr/Re^2)\cos\alpha]$ and $Nu/Nu_0 = g[(Gr/Re^2)\cos\alpha]$ where $C_f$ is the local skin friction coefficient, $Gr$ the Grashof number, $Re$ the Reynolds number, $Nu$ the Nusselt number and $\alpha$ is the angle between the free-stream flow direction and the gravitational acceleration vector. $C_{f0}$ and $Nu_0$ are values for a horizontal plate.

An example is given for the case of a free-stream velocity of 4 mps, a free-stream static temperature of 273 K and a plate temperature of 567 K. At a station 0.5m from the leading edge, $C_f/C_{f0}$ was shown to vary over the range $0.54 < C_f/C_{f0} < 1.34$, depending on the inclination of the plate.

E. K. Parks, USA

**5108. Epstein, M., On the solution of the laminar boundary-layer equations,** *J. Aero/Space Sci.* **26,** 1, 58–59 (Readers' Forum), Jan. 1959.

Author transforms Prandtl's boundary-layer equation into a nonlinear integral equation for the shearing function and solves this equation by iteration. He chooses the first approximation in such a way that it satisfies the boundary conditions and has the correct asymptotic behavior at infinity. Application to the shearing stress distribution on a circular cylinder and other examples shows good agreement with the exact solutions.

K. Pohlhausen, USA

**5109. Witting, H., Influence of streamline curvature on the stability of laminar flows** (in German), *Arch. Rational Mech. Anal.* **2,** 3, 243–283, Dec. 1958.

Author studies the stability of a laminar flow affected by two-dimensional waxy disturbances with respect to Taylor-Görtler vortices in the depressions of the waves. There results an eigenvalue problem which, when solved, shows that the wavy laminar flow becomes unstable at a large curvature of the wave. In particular it is shown that, for a plane Poiseuille flow, the instability occurs when the amplitude of the wave has a critical value of the order of $10^{-4}$ of the boundary-layer thickness. This may help explain the transition process from laminar to turbulent flow. The laminar flow, at first unstable to the Tollmien's waves, becomes unstable with respect to Taylor-Görtler vortices, when the amplitude of the Tollmien's waves is critical. For three-dimensional flow the transition to turbulent flow occurs owing to instabilities of higher order. Of course this mechanism must be verified by experiment. G. Noseda, Italy

**5110. Mark, H. J., Rapid calculations for boundary-layer transfer using wedge solutions and asymptotic expansions,** *J. Fluid Mech.* **5,** 3, 460–486, Apr. 1959.

Paper (a) employs method of Meksyn for calculating constant-property laminar convective heat-transfer-coefficient distribution

over bodies of uniform temperature but arbitrary shape, and (b) derives asymptotic expansion for Nusselt number of wedges in terms of Prandtl number. Method of (a) has advantage over all previous methods considering only one differential equation in that accuracy, already good in zero'th approximation, is capable of improvement by consideration of further terms of a series; coefficients of these terms have, however, not yet been calculated. Author indicates that extensions are possible to nonuniform temperature, finite mass transfer rates, and compressible flow. Reviewer believes this is an important paper.

D. B. Spalding, England

**5111. Curle, N., Some details of the quintic profile for use in the Polhausen-type of boundary-layer calculation,** *Aero. Res. Counc. Lond. Curr. Pap.* 391, 8 pp., 1958.

Polhausen method is worked out using 5th-degree polynomial. Additional boundary condition satisfied is $u'''(0) = 0$. Results obtained are not in any way intermediate to those of Polhausen (4th-degree polynomial) and Schlichting-Ulrich (6th-degree polynomial). Separation is at $\lambda = -20/3$ and no stagnation point solution at all is found. Method cannot be used, therefore, for accelerating flow. Universal functions $g(\lambda)$ and $b(\lambda)$ are tabulated, and method is applied to case of linearly retarded free-stream velocity. Agreement with Howarth's exact solution to within 6% is found.

L. Mack, USA

**5112. Vaglio-Laurin, R., Laminar heat transfer on blunt-nosed bodies in three-dimensional hypersonic flow,** WADC TN 58-147 (Polyt. Inst. Brooklyn, Dept. Aero. Engng. Appl. Mech.; ASTIA AD 155 588), 29 pp., May 1958.

It is shown that in the presence of highly cooled surfaces and of moderate Mach numbers at the outer edge of the boundary layer, the crossflow in the boundary layer is negligible even for large transversal pressure gradients. From this consideration a rapid method for estimating the heat transfer over the nose portion of arbitrary three-dimensional blunt bodies in hypersonic flight is derived. The approximate results thus obtained compare very favorably with known exact solutions. The method in question reduces the boundary-layer problem to the determination of the inviscid streamline pattern on the surface of the body; a procedure for obtaining these streamlines from a pressure distribution measurement is outlined.

From author's summary by W. Squire, USA

**5113. Gupta, A. S., Advancement of a compressible heat-conducting fluid over an infinite flat plate** (in English), *ZAMM* **37,** 9/10, 349-353, Sept./Oct. 1957.

The unsteady flow of a fluid advancing over a flat plate with a constant velocity $u_1$ is investigated. The flow exists for $x < u_1 t$, where $x$ is along the length of the plate and $t$ the time. Approximate velocity and temperature distributions are found for similarity solutions with $(u_1 t - x)$ as the axial length, using the method of Meksyn. The unsteady effect appears primarily in the variable $(u_1 t - x)$ which replaces $x$ as the typical length in the Blasius solution of steady flow.

H. G. Lew, USA

**5114. Ray, M., Boundary layer in a perfect gas over a flat plate under pressure gradient,** *Bull. Calcutta Math. Soc.* **49,** 3, 133-138, Sept. 1957.

Boundary-layer flow over a flat plate is studied under the assumptions: steady flow, perfect gas, uniform main stream, variable pressure along the plate, viscosity inversely proportional to density. Further simplification is achieved assuming density and temperature variation from main stream values to be small. Solution is given, layer thickness is computed. Method gives rise to infinite $\partial u/\partial y$ on the plate.

G. Moretti, USA

**5115. Carrier, G. F., and Chang, C. T., On an initial value problem concerning Taylor instability of incompressible fluids,** *Quart. Appl. Math.* **16,** 4, 436-439 (Notes), Jan. 1959.

The problem of Taylor instability concerns the behavior of a disturbance of the interface separating two bodies of fluid when the system is accelerated from the lighter toward the heavier fluid. Present paper includes effects of viscosity and surface tension, and studies the growth of the disturbance when an initial surface disturbance is specified.

The solution of the problem yields a "cut-off" wave number above which disturbances are not amplified. The value of this wave number is the same as that obtained by Bellmann and Pennington [*Quart. Appl. Math.* **12,** 151-162, 1954] for ordinary exponential instability.

J. T. Stuart, England

**5116. Cramer, K. R., A generalized porous-wall "Couette-type" flow,** *J. Aero/Space Sci.* **26,** 2, 121-122 (Readers' Forum), Feb. 1959.

Couette flow with constant, uniformly distributed suction or blowing is treated analytically and calculated data are presented in terms of dimensionless velocity profiles parametric in the suction velocity (or blowing velocity) parameter. Author again calls attention to fact that analysis of Couette flow with pressure gradient can be applied to hydrodynamic theory of bearing lubrication.

R. M. Drake, Jr., USA

**5117. Liu, V. C., On the separation of gas mixtures by suction of the thermal-diffusion boundary layer,** *Quart. J. Mech. Appl. Math.* **12,** 1, 1-13, Feb. 1959.

This paper discusses the formation and characteristics of the boundary layer of thermal diffusion existing along the surface of a heated wall over which flows a mixture of gases of unequal molecular weights. Due to the existence of a temperature gradient in the boundary layer, a thermal-diffusion flux is set up such that the lighter gas tends to move to the hotter region and the heavier gas to the colder region. When the temperature of the surface is maintained at a much higher level than that of the free stream, the concentration of the lighter gas increases monotonically from its free-stream value at the outer edge of the boundary layer to a maximum value at the plate.

A limiting ratio of suction against free-stream velocity for the maintenance of laminar flow is derived. For the case of isothermal incompressible flow the limiting wall suction velocity $v_0$ is of the order of magnitude of 0.01% of the free-stream velocity. An estimation of the rate of attaining the equilibrium-concentration profile is also made. The time to attain steady state approaches $\nu_0/(\pi V_0)^2$, where $\nu_0$ is the wall kinematic viscosity. Formulas are derived for the diffusive separation factor, but practical values or cascades are not calculated.

From author's summary by C. F. Bonilla, Puerto Rico

**5118. Lelte, R. J., An experimental investigation of the stability of Poiseuille flow,** *J. Fluid Mech.* **5,** 1, 81-96, Jan. 1959.

Stability characteristics of axially symmetric laminar flow were experimentally observed in a smooth pipe up to $Re = 20{,}000$. Small, nearly axially symmetric disturbances were superimposed by longitudinal oscillation of a thin internal sleeve at the pipe wall. Radial and longitudinal surveys using a hot-wire anemometer revealed: (1) flow is stable to small disturbances up to $Re = 13{,}000$; (2) in general, decay rates were found to agree with theoretical analysis of Corcos and Sellar [*J. Fluid Mech* **5,** 1959]; (3) to first-approximation disturbance propagation velocity is independent of distance downstream from source; (4) small disturbances decay at all $Re$ investigated and that large disturbances are unstable. Author concludes that, for fixed $Re$, disturbance of marginal stability existed but was not observed directly.

M. S. Macovsky, USA

## Turbulence

(See Revs. 5118, 5223)

## Aerodynamics

(See also Revs. 5169, 5198, 5226, 5265)

**5119. Henshall, B. D., and Cash, R. F., Observations of the flow past a two-dimensional 4 per cent thick biconvex aerofoil at high subsonic speeds,** *Aero. Res. Counc. Lond. Rep. Mem.* 3092, 17 pp., 1958.

Flow photographs and detailed pressure distributions for a 4 per cent thick circular-arc biconvex airfoil at high subsonic speeds and high incidences have been analyzed and the divergence boundary (defining the onset of separation effects) for the airfoil determined. Emphasis was placed on the transition from leading-edge to shock-induced separation as the free-stream Mach number was raised at a fixed incidence. From authors' summary

**5120. Henshall, B. D., and Cash, R. F., Observations of the flow patterns of a two-dimensional 4 per cent thick biconvex aerofoil at $M_0 = 1.40$ and 1.63,** *Aero. Res. Counc. Lond. Rep. Mem.* 3093, 9 pp., 1958.

Direct-shadow and schlieren photographs and pressure distributions of the flow past a two-dimensional 4 per cent thick biconvex airfoil for a limited range of incidences at Mach numbers of 1.40 and 1.63 are presented. Shock-induced boundary-layer separation at the trailing edge of the airfoil was present at $M_0 = 1.63$ with transition-free boundary layers but was absent up to 5 deg incidence at $M_0 = 1.63$ with transition-fixed boundary layers and up to 6 deg incidence at $M_0 = 1.40$ with transition-free boundary layers. From authors' summary

**5121. Sheppard, K. M., and Thomson, K. D., A shock-expansion theory applicable to wings with attached shock waves,** *Aero. Res. Counc. Lond., Curr. Pap.* 392, 17 pp., 1958.

Paper describes calculation of surface pressures on finite wings in definitely supersonic flows. The shock-expansion method is modified to include certain three-dimensional effects. The wing surface is replaced by a polyhedron whose edges and faces are separately analyzed. Portions of the wing that are not two-dimensional are treated by use of characteristic calculations in the plane faces of the polyhedron. Tip regions on lifting wings must be excluded. Authors suggest that method applies to wings for which $\tan \Lambda < (M^2 - 1)^{1/2}/3$; $\Lambda$ is leading edge sweepback angle.

F. B. Fuller, USA

**5122. Sirazetdinov, T. K., Computation for wings with a curvilinear axis** (in Russian), *Trudi Kazansk. Aviats. In-ta* **31**, 51-64, 1956; *Ref. Zh. Meko.* no. 4, 1958, Rev. 3857.

Author examines calculation for the distribution of the circulation over the span of a wing of large elongation with a curvolinear central line. General formulas are also presented for the calculation of aerodynamic coefficients on the transcritical angles of attack. The flow, to start with, is considered to be incompressible, and subsequently, a computation is made in accordance with the linear theory for the influence of compressibility at transonic velocities. The coefficient of lifting force for any section is taken to be the function of the profile form, the actual angle of attack and the local angle of the sagittal shape of the central line. This enables the characteristics of the section of a wing of finite span to be linked with the characteristics of an infinitely long slipping wing. A special feature of the calculation is the method of computing the inductive velocities. When this is being done in a section situated at a distance $\zeta = z$ from the plane of symmetry of the wing, the last, on the portion of the span $z - \Delta_1 < \zeta < z + \Delta_2$ is replaced by the carrying vortex surface $\sigma$ with even distribution of vortex action along the chord. On the remaining portion of the wing's span a single carrying vortex, coincident with the central line, is substituted. The mean value of the inductive velocity on section $\zeta = z$ is calculated from the carrying vortex surface $\sigma$ and the vortices running off it. The inductive velocity of the carrying and free vortices of the remaining part of the wing is calculated from the mean point of the section. By this means author avoids the infinite values of these velocities when the central line is bent. The width of the vortex strip $\sigma$, equal to $\Delta_1 + \Delta_2$, is selected in such a way that the nucleus of the integral equation for the circulation should alter continuously in relation to the span. The equation is solved by a method similar to the known Multhopp method.

G. F. Burago

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5123. Schepisi, G., The practical calculation of the load distribution on aircraft,** *AGARD Rep.* 110, 21 pp., May 1957.

The object of this paper is to collect in a convenient form the calculations for the load distribution on an aircraft. The mass forces are first derived in terms of the rigid and elastic degrees of freedom of aircraft motion, using the aircraft dynamical equations.

The equations are solved for prescribed steady-flight conditions (equilibrium equations) in a symmetrical case, with the aid of the component aerodynamic distributions, which can be obtained separately by aerodynamic theory. An outline of the earlier method is also given.

Finally, the solution is attempted for unsteady flight conditions, with the help of unsteady lift functions.

From author's summary

**5124. Miele, A., Interrelationship of calculus of variations and ordinary theory of maxima and minima for flight mechanics applications,** *ARS J.* **29**, 1, 75-76 (Tech. Notes), Jan. 1959.

**5125. Cole, J. D., Lift of slender nose shapes according to Newtonian theory,** Rand Corporation, P-1270, 17 pp., Feb. 1958.

Newtonian theory for slender bodies of thickness ratio $\delta$ at zero incidence predicts a pressure coefficient $0(\delta^2)$ and similar flows for the Newtonian similarity parameter $N$

$$N = \frac{\gamma + 1}{\gamma - 1} \frac{1}{M_\infty^2 \delta^2}, \qquad M_\infty = \text{Mach number}$$

fixed. The pressure on the surface turns out to be independent of $N$. In this memorandum the analogous calculation is carried out for small angles of attack $\alpha$ where the part of pressure coefficient which contributes to lift is $0(\delta\alpha)$. The problem is considered as a small perturbation on the solution at zero incidence. A complete solution is found and the results for lift coefficient $C_L$ and center of pressure $x_{c.p.}$ are

$$\frac{dC_L}{d\alpha} = \frac{4}{3}\left\{1 + F'(1)\int_0^1 F(x)\,dx\right\}$$

$$x_{c.p.} = 1 - \frac{1}{\dfrac{dC_L}{d\alpha}} \frac{4}{3}\int_0^1 F(x)\,dx$$

where the body shape is given by $r$ = radius $= \delta F(x)$, $F(0) = 0$, $F(1) = 1$. From author's summary

**5126. Mottard, E. J., Effect of convex longitudinal curvature on the planing characteristics of a surface without dead rise,** *NASA Memo* 1-25-59L, 34 pp., Feb. 1959.

The effects of convexity were to increase the wetted length-beam ratio (for a given lift), to decrease the lift-drag ratio, to

move the center of pressure forward, and to increase the trim for maximum lift-drag ratio as compared with values obtained for a flat surface. The effects were greatest at low trims and large drafts. The maximum negative lift coefficient based on the model beam obtainable with a ratio of the radius of curvature to the beam of 20 was – 0.02. The effects of camber were greater in magnitude for convexity than for the same amount of concavity.

From author's summary

**5127. Reeder, J. P., The effect of lift-drag ratio and speed on the ability to position a gliding aircraft for a landing on a 5,000-foot runway,** *NASA Memo* 3-12-59L, 10 pp., Apr. 1959.

Flight tests were made to determine the capability of positioning a gliding airplane for a landing on a 5000-foot runway with special reference to the gliding flight of a satellite vehicle of fixed configuration upon reentry into the earth's atmosphere. The lift-drag ratio and speed of the airplane in the glides were varied through as large a range as possible.

From author's summary

**5128. Ribner, H. S., On the lift and induced drag associated with large downwash angles,** *Univ. Toronto, Inst. Aerophys.* TN 19, 15 pp. + 6 figs., Jan. 1958.

A momentum balance has been carried out for an arbitrary wing disturbance pattern enclosed in a control surface. The lift and induced drag have been evaluated without the assumption that the downwash angle or disturbances are small.

The main finding is that the entire induction effect of the wake can be represented by a "generalized" induced drag vector *parallel to the asymptotic trailing vortex direction.* The magnitude is measured by the kinetic energy per unit length of the *inclined* trailing system. The component in the lift direction is negative, and represents a nonlinear contribution to lift. A possible similar behavior of the parasite drag is touched on.

The Jones low-aspect-ratio wing theory has been reformulated to apply to large angle of attack; it gives complete agreement with the momentum-balance results.

The asymptotic inclination of the trailing vortex system and hence of the induced drag depends markedly on whether the sheet remains plane (an unstable condition) or rolls up into a vortex pair. The magnitude of the inclined drag is simply determined for the planar case, but some uncertainty remains for the more realistic rolled-up-pair case. From author's summary

**5129. Laurmann, J. A., The effect of slip on induced pressures,** *J. Aero/Space Sci.* **26**, 1, 53–54 (Readers' Forum), Jan. 1959.

**5130. Casaccio, A., Theoretical pressure distribution on a hemisphere-cylinder combination,** *J. Aero/Space Sci.* **26**, 1, 63–64 (Readers' Forum), Jan. 1959.

**Book—5131. Goroshchenko, B. T., The dynamics of flight of an aeroplane; Manual for aviation higher educational institutions** (in Russian), Moscow, Oborongiz, 1954, 336 pp. 9r, 35k; *Ref. Zh. Mekh.* no. 4, 1958, Rev. 3906.

Book is a manual for aviation higher educational institutions. It is devoted fundamentally to the investigation of the irregular motion of the center of gravity of an airplane when executing different flight maneuvres. The book is divided into two parts. In the first part there is an exposition of the problems of the rectilinear motions of an airplane, while in the second, the maneuvring of the airplane along curvolinear trajectories. In the introduction the author gives an historical review of the development of "The dynamics of flight of an airplane," commencing with the scientific works of N. E. Zhukovskii and then of V. P. Vetchinkina, as founders of the principles of this aviation study. The work done by Soviet scientists is briefly described. The investigation of the separate characteristic maneuvres is usually approached by a detailed exposition of the physical aspects of the phenomenon. Then inferences are drawn, and descriptions given of different methods of calculation for the determination of the quantitative characteristics of the evolution being examined; and, finally, an analysis is carried out regarding the influence of the constructional parameters and conditions of flight on the maneuvring qualities of the airplane. Much attention is devoted to the description and analysis of the process of piloting of an airplane. In regard to the maneuvres being investigated, and to make their exposition more of an object lesson, notes are given of the changes with time of the parameters of the airplane's motion and of the angles of declination of the rudder-flaps, obtained in flights on a training fighter-plane. In chapter I the acceleration and drag of the airplane are investigated along rectilinear trajectories. The possibilities are analyzed of improving some of the characteristics of acceleration and drag by means of using wave-like trajectories. Chapter II is devoted to the take-off of an airplane and chapter III to the landing. In addition to describing the various methods of calculation and the influence of aerodynamic and constructional parameters, means of improving the take-off and landing characteristics of modern airplanes are investigated; in particular, the use of boosters when taking off, and reversed pull and air-braking when landing. In chapter IV the analytical method of V. P. Vetchinkin is detailed for the calculations of the irregular rectilinear motions of an airplane when landing and when taking off.

In chapter V, which opens the second part of the book, an examination is made of the basic concepts and of the general position which are essential for the study and analysis of the curvolinear motion of an airplane. The elements of control of an airplane in curvolinear flight are investigated. Chapter VI deals with methods of calculation and analysis of different types of banking in an airplane: regular and irregular, coordinated and with slip, regular, limiting, and so forth. The influence of altitude of the flight, of the air compression and of the constructional parameters of the airplane on the characteristics of the banking, etc. is investigated. At the end of the chapter an analysis is given of the veering of airplanes in formation flying. In chapter VII an investigation is made of the evolutions of diving and climbing, mainly performed in a vertical plane. Chapter VIII is devoted to the evolutions: battle "peeling off," oblique attack and spirals. In chapter IX the following air-evolutions are scrutinized: the Nestor loop, single and double turns, the half-loop with a turn. The conditions governing their characteristics are stated; these are essential for the correct performance of these figures. Examples are given to show how the energy method can be applied to the analysis of the motion of an airplane. G. S. Kalachev

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5132. Hacker, T., On the longitudinal stability of an aircraft in case of repeated disturbances** (in Roumanian), *Studii Si Cercetari Mecan. Appl.* **10**, 1, 69–76, 1959.

Problem is discussed with the aid of the theory of motion stability under continuously acting disturbing forces. The procedure is then applied to the case of repeated vertical gusts. For this case an approximate formula is deduced for determining an allowable bound of the gust velocity in terms of the coefficients of the disturbed motion equation. A. Petre, Roumania

**5133. Roth, W., Suitable roll geometry for the rolling of very thin sheets** (in German), *Ing.-Arch.* **26**, 5, 333–337, Oct. 1958.

The problem concerns the exact theoretical shape of the rolls for two-high foil mills. The solution to the problem proceeds from the assumption that the necessary conditions for rolling of thin foil of uniform thickness culminate in a uniformly distributed force along the line of contact. The solution is rigorous and involves a potentially time-consuming, iterative process in a convergent series, although the author demonstrates that calculation can be

relatively brief for rolls of practical proportions. This solution should prove useful to those who wish to use only two rolls for making foil.

L. V. Colwell, USA

**5134. Moul, M. T., and Brennan, Teresa R., Approximate method for calculating motions in angles of attack and sideslip due to step pitching-and yawing-moment inputs during steady roll,** *NACA TN* 4346, 19 pp. + 4 tables + 5 figs., Sept. 1958.

The approximate method assumes a constant rate of roll and uses the four equations of motion which represent the equilibria of pitching and yawing moments and the equilibria of normal and side forces. The general expressions of the coefficients of the characteristic equation and the responses in angle of attack and sideslip to unit step inputs in yawing and pitching moments are presented.

For the case of a swept-wing fighter-type aircraft flying with Mach number 0.7 at 32,000-ft altitude the simplified analytical solutions are compared with results of accurate analog computer calculations which treat the rolling motion produced by an aileron step input as a fifth degree of freedom. The time histories of angle of attack and sideslip calculated by both methods agree sufficiently so that the simplified method can be used for estimates of aircraft design loads. Similarly good agreement can be expected if the approximate method uses a suitable average rate of roll and if no rolling divergence is encountered.

G. W. Braun, USA

**5135. Kalinin, S. V., On the stability of airplane motion with autopilot** (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 4, 114–117, Apr. 1958.

Author extends earlier work on the problem of stability of autopilot-airplane systems. He restricts himself to the control of the angle of yaw of the airplane, the rudder angle being the controlling variable. The rudder velocity is assumed to be a nonlinear servomotor function of the control valve opening (or equivalent device) where this opening is a linear combination of the yaw error, the derivative of this error and the rudder angle. The dependence on rudder angle is accomplished through feedback from this angle to the valve. It is assumed that pulsations arise in the servomotor, corresponding to sinusoidal terms in the servomotor functions. Two types of servomotor functions are treated. The one is proportional to the cube of valve opening, and the other is a linear combination of this term and the valve opening itself. Author determines the effect of the forcing oscillation on system stability. He employs a function of Lyapunov,and uses the technique of averaging periodic coefficients, assuming that system variables change slowly during one period of the forcing oscillation. By Lyapunov's stability theorum, he relates stability of the autopilot-airplane system to the magnitude of the forcing oscillation.

The approach of this paper follows the Russian school. The choices of the servomotor function appear to have been made for mathematical convenience, but are reasonable in the light of practical experience. The forcing function is assumed to affect only the cubic term in the servomotor function.

R. Oldenburger, USA

**5136. Truckenbrodt, E., and Feindt, E. G., Stall characteristics of delta wings in incompressible flow** (in German), *Z. Flugwiss.* **6**, 4, 97–102, Apr. 1958.

Stall characteristics of delta wings in symmetrical incompressible flow have been investigated at Reynolds numbers up to $Re = 1.7 \times 10^6$. Four delta wings of different aspect ratios employing NACA 0012 profile-section and one delta wing with NACA 0005 profile-section have been tested. The spanwise distribution of the critical angle of incidence at which flow separation begins was obtained from wake survey. Experience has shown that the separation of flow over delta wings of low aspect ratios occurs at considerably greater angles of incidence than over delta wings with high aspect ratios, and that the flow over a thick airfoil is less prone to separate than that over a thin one. Separation begins at the wing tips, progressing toward the plane of symmetry of the wing with increasing angle of incidence, The final results are compared with measurements obtained of pressure distribution.

From authors' summary by B. Regenscheit, Germany

**5137. Sternfield, L., A simplified method for approximating the transient motion in angles of attack and sideslip during a constant rolling maneuver,** *NACA Rep.* 1344, 11 pp., 1958.

The transient motion in angles of attack and sideslip during a constant rolling maneuver has been analyzed. Simplified expressions are presented for the determination of the pertinent modes of motion as well as the modal coefficient corresponding to each mode.

Calculations made with and without the derivatives $C_{Y_\beta}$ (side force due to sideslip) and $C_{L_\alpha}$ (lift-curve slope) indicate that although these derivatives increase the total damping of the system they do not markedly affect the transient motion.

From author's summary by Irmgard Flugge-Lotz, USA

**5138. Stromilov, V. M., Approximate method of solution of the characteristic equation for the lateral movement of an aeroplane** (in Russian), *Trudi Mosk. Aviats. In-ta* no. 75, 5–11, 1957; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 2780.

An approximate method is submitted for the determination of the roots of the characteristic equation of a system of equations for the lateral movement of an airplane, applicable to cases of airplanes of ordinary construction; two real roots are obtained, and two complex combined roots. The method put forward reduces the amount of calculation work and indicates in a clear manner the dependence of the roots of the characteristic equation on the aerodynamical and constructional parameters of the airplane.

Yu. I. Sneshko

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5139. Gillis, C. L., and Mitchell, J. L., Determination of longitudinal stability and control characteristics from free-flight model tests, with results at transonic speeds for three airplane configurations,** *NACA Rep.* 1337, 28 pp., 1957.

**5140. Huggett, D. J., The ground effect on the jet flap in two dimensions,** *Aero. Quart.* **10**, 1, 28–46, Feb. 1959.

The effect of the ground on a jet-flap airfoil in two-dimensional flow is investigated, with particular reference to the conditions under which the jet flow hits the ground. Experimental results are included for the change of lift and pitching moment for 58.1° and 31.4° jet-deflection angles, with a limited investigation into the downwash changes behind the model with a 58.1° jet-deflection angle. It is concluded that there is a definite limit to the jet coefficient, which may be used at any ground position, if the problems of a sudden change of lift and pitching moment are to be avoided. There remains the unresolved difficulty of the downwash variation at the tail arising from the presence of the ground.

A provisional assessment of the performance of a jet-flap aircraft near the ground is made, and shows that for a 58.1° jet-deflection angle there is a definite limit to the minimum flying speed for any given distance of the wing from the ground.

The fact that there appears to be a maximum pressure lift coefficient, independent of the jet-deflection angle, which may be obtained at any ground position, is investigated. An attempt is made to provide a theoretical explanation of this on the basis of the critical condition of blockage underneath the airfoil.

From author's summary

## Vibration and Wave Motion in Fluids

(See Revs. 5115, 5319)

## Fluid Machinery

(See also Revs. 4959, 5269)

**Book—5141. Lazarkiewicz, Sz., and Troskolanski, A. T., Rotodynamic pumps [Pompy wirowe],** Warszawa, Państwowe Wydawnictwa Techniczne, 1959, 553 pp. + 13 plates. Zl. 80.

A modern and well-elaborated treatise on impeller pumps, the first of such scope in the Polish language; it can be favorably compared with works by H. Addison, C. Pfleiderer and A. J. Stepanoff. Book covers all types of rotodynamic pumps: centrifugal, screw-type impeller, half-axial and axial-flow impeller types. Contents are very extensive: basic theory of vortex pumps, balance of energy, liquid flow through a rotor, dynamic similarity of pumps, shape of rotors with plain vanes and of double curvature, impeller blades, inlets and outlets, structural elements, pressure and cavitation, characteristics, efficiency regulation, drives, examples of design, self-priming pumps, ejectors, pump and model tests, installation and maintenance. Tables and indices conclude this perfectly edited book.

The text is lucid and can be easily translated with the aid of a multilingual index at the end of the book. State Technical Publishing House of Poland can be praised for the print and design of very high quality. There can be no doubt that this useful book will be translated into other languages.

S. Kolupaila, USA

**5142. Schilhansl, M. J., A simple approach to an approximate two-dimensional cascade theory,** *J. Appl. Mech.* **25,** 4, 607–612, Dec. 1958.

The approach is based on the so-called singularity method, in which the blade sections are replaced by sheets of vortexes, sources and sinks, and the flow induced by these singularities is calculated. The condition that the flow must be tangential to the blade surface, sometimes termed the tangency condition, leads to a relation between the geometrical shape of the blade sections, the cascade parameters, and the singularity distributions along the mean camber lines. As soon as these distributions are known, the pressure distribution and the lift may be determined. A short simple method of calculating the velocities induced by the singularities is described.

R. C. Binder, USA

**5143. Seippel, C., Three-dimensional flow in multi-stage turbines,** *Brown Boveri Rev.* **45,** 3, 99–107, Mar. 1958.

Theoretical study is presented of rotational and irrotational flow in axial turbomachinery. The analysis of a multistage machine is based on the following assumptions: (a) The flow is frictionless and incompressible. (b) The flow is rotationally symmetrical. (c) After each stage the velocity field is the same as at the inlet to the stage. All changes in velocity in the stationary and moving blading are just of equal magnitude and opposite sign. (d) The blades are capable of exerting forces in the tangential and axial directions only.

Several curves are given which make the calculation very simple for the special case of constant rotation and constant axial velocity. Author agrees that as the analysis assumes frictionless flow the losses must be determined experimentally in order to make the analysis applicable for design purposes.

K. Pilarczyk, USA

**5144. Troskolanski, A. T., Fundamental problems of the dynamic similarity of rotodynamic hydraulic machinery** (in Polish), *Przeglad Mech.* **17,** 11, 495–501, 1958.

Author develops the theory of dynamic similarity of rotodynamic hydraulic machinery and, using dimensional analysis, derives a dimensionless expression for specific speed, the same for rotors of turbines and impellers of pumps: $n_{sf} = 1000\, nQ^{0.5}/60\,(gH)^{0.75}$ where $n$ is in rpm. This unique and universal dimensionless number can be applied internationally for normalization and model tests of turbines and pumps. Paper deserves attention.

S. Kolupaila, USA

**5145. Hunziker, R. R., Hydrodynamic influence of the propeller on a deeply submerged submarine,** *Inter. Shipbldg. Prog.* **5,** 44, 166–177, Apr. 1958.

Neglecting viscous effects, the force acting on the hull due to a rear propeller is calculated by approximating the propeller by a sink disk, taking the sink density to be a given constant. The force is obtained as an integral of the in-flow velocity induced by the hull at the disk. Result agrees with those of earlier investigators obtained under more restricted conditions. Numerical results for prolate ellipsoid hulls are presented.

S. B. Berndt, Sweden

**5146. Patraulea, N. N., Tandem opposing airscrews** (in Roumanian), *Studii Si Cercetari Mecan. Appl.* **10,** 1, 41–67, 1959.

Author studies the flow induced by a propulsive system formed by two coaxial airscrews, their respective pitch angles being so adjusted as to make the first to yield a positive thrust while the second works as a windmill. In this way, between the two airscrew disks a tube formed by free annular vortices is created, whose sense is such as to induce higher rates of flow through the airscrews than in the case of a single airscrew having the same total loading. Downstream of the second airscrew, the slipstream undergoes a dilatation instead of a contraction, so that air velocities are higher between the two airscrews than outside them. Approximate (linearized) theoretical treatment is given and also a more exact solution through Helmholtz's method for an equivalent plane flow. It is shown that, by using the proposed configuration, the aerodynamic characteristics of the system can be improved, e.g., the figure of merit can be increased indefinitely. Friction losses are then computed and are shown to have a considerable influence. However, a substantial increase in the momentum of the air jet between the two airscrews is shown to be easily obtainable. By placing an airfoil in this jet, high lift can be realized through the deflection of the jet with moderate $C_z$ and with low power consumption. This constitutes a substantial improvement of the VTOL scheme of deflected slipstream. Other interesting features of the proposed system are also discussed.

L. Dumitrescu, Roumania

## Flow and Flight Test Techniques and Measurements

(See also Rev. 5068)

**Book—5147. Messersmith, C. W., Warner, C. F., and Olsen, R. A., Mechanical engineering laboratory,** 2nd edition, New York, John Wiley & Sons, Inc., 1958, 179 pp. $5.50.

This manual is organized as an instruction handbook for the measurement of physical properties, fluid flow, power and apparatus performance. The instruction is well ordered and contains many code references. It includes required techniques for setup, instrumentation, test procedure, and report writing. This edition continues in the excellent tradition of the first edition

[AMR 4 (1951), Rev. 692] with effective organization of the material on temperature measurement, flow measurement, power measurement, and fluid pumps. The chapters on physical properties, analysis of products of combustion and refrigeration have been expanded. The result is a well-rounded, up-to-date text.

E. G. Allen, USA

**5148. Soroko-Novitskii, V. I., Investigation of the influence of inaccurate manufacture of calibrated orifices (nozzles) on the discharge of liquid through them and the determination of tolerances in their accurate preparation and calibration** (in Russian), Trans. of the Chair of Automobiles and Tractors, Vses. Zaochn. Machinostroit-In-ta no. 1, 89–94, 1957; *Ref. Zh. Mekh.* no. 4, 1958, Rev. 4112.

Making use of the d'Arcy-Weisbach formula, author obtains an equation which enables him to determine the permissible tolerance in the dimensions of a nozzle orifice so that the discharge of fuel from the nozzle shall not exceed the limits laid down by the conditions of economy of working of the motor. The method proposed by the author furnishes the possibility of determining the necessary accuracy in manipulating and in adjusting the other elements in the delivery devices of the carburetor, of establishing the necessary accuracy for the positioning of the atomizer and diffuser and of explaining the regulating deficiencies of the carburetor because of inaccuracies in manufacture. Yu. A. Lashkov

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5149. Listov, A. M., and Drozdov, V. A., A differential thermoelectroanemometer** (in Russian), Soobshch. Vses. N.-i. In-ta Zh.-d. Str-va i Proektir. no. 66, 18 pp., 1955; *Ref. Zh. Mekh.* no. 11, 1957, Rev. 12888.

**5150. Silsby, N. S., External interference effects of flow through static-pressure orifices of an airspeed head at several supersonic Mach numbers and angles of attack,** *NASA Memo* 2-13-59L, 14 pp., Mar. 1959.

Wind-tunnel tests have been made to determine the static-pressure error resulting from external interference effects of flow through the static-pressure orifices of an airspeed head at Mach numbers of 2.4, 3.0, and 4.0 for angles of attack of 0°, 5°, 10°, and 15°. Results were obtained for flow out of the orifices for a range of mass flow from 0 to about $4 \times 10^{-6}$ slugs per second.

From author's summary

**5151. Laycock, G. H., A constant liquid flow device,** *Brit. J. Appl. Phys.* **9,** 8, 333–335, Aug. 1958.

Paper presents a novel method for achieving a flow device with characteristics independent of viscosity and illustrates its use with the design and verification of a specific instrument. Basic approach involves a series arrangement of two flow devices whose head-discharge relations vary in opposite fashions with viscosity. In the case exemplified, a pipeline orifice, sized and proportioned so as to operate in the range where the coefficient decreases with increase in Reynolds number (i.e. viscosity), is followed by a bank of fine tubes, in which the flow is laminar-viscous. Test of the model built closely verified the design and yielded a constant ($\pm 0.6\%$) flow, for a given pressure drop, over a threefold range in viscosity. Author suggests an alternative method in which two orifices may be secured in the pipeline in series. The one orifice is proportioned so that the flow occurs in the range where coefficient increases with Reynolds number and the other so that coefficient decreases. J. M. Robertson, USA

**5152. Spikes, R. H., and Pennington, G. A., Discharge coefficient of small submerged orifices,** Instn. Mech. Engrs., Prepr., 5 pp., 1958.

Paper deals with the variation in discharge of small submerged orifices used in aero-control systems. It is shown that the effects of cavitation can be high and can cause changes of discharge coefficient greater than those associated with Reynolds number when the flow is turbulent.

The orifice length was altered and it was found that the most unstable conditions occurred when the length was half the diameter.

Further tests were carried out with inlet chamfer in order to determine the optimum angle and depth of chamfer for cavitation suppression. From authors' summary

**5153. Solokhina, E. V., Investigation of the parameters of a flow in the axial clearance of a gas turbine** (in Russian), *Trudi Mosk. Aviats. In-ta* no. 63, 61–81, 1956; *Ref. Zh. Mekh.* no. 4, 1958, Rev. 3974.

Results are given of the experimental investigation of the field of velocities in the axial clearance of a gas turbine when the number of revs per minute is increased to 12,000. An approximate calculation is put forward for the field of velocities in the axis of the inter rim clearance of the turbine according to the spacing of the jet apparatus. The calculation is based on the use of data for the track behind the cylindrical rod and the separate aspects of the theory of turbulent streams. To carry out the calculation, knowledge of two experimental constants is imperative. The results of the calculation agree satisfactorily with experiment.

V. I. Vasil'ev

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5154. Baier, M.-Ya., A conforming method for profiling of the blades of radially axial turbomachine** (in Russian), *Energomashinostroenie* no. 4, 18–19, 1957; *Ref. Zh. Mekh.* no. 4, 1958, Rev. 3980.

Author states that "the method of conforming reflections for the profiling of the blades of radially axial turbomachines . . . . , contradictory in its initial theoretical premises, out of step with the scientific and technical concepts of empiricism, can not be recognized as a scientific method and, consequently, should be rejected." (The method of conforming reflections is the author's designation for the known method of investigation of a two-dimensional flow in a layer by means of reflecting it on to a plane surface; see, for instance, Lamb, "Hydrodynamics," Moscow, Gostekhizdat, 1947.) The publication of this clearly illiterate work even in "the category of discussion" evokes surprise.

G. Yu. Stepanov

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5155. Kondryakov, I. K., and Bazovskii, V. N., The dependence of the volume of an interlaminated chamber on the angle of rotation of a rotor of an expansion machine of the revolving type** (in Russian), *Trudi Leningrad Tekhnol. In-ta Kholodil'n. Prom-sti* **13,** 135–139, 1957; *Ref. Zh. Mekh.* no. 4, 1958, Rev. 3987.

Formulas are proposed for the determination of the volume of an interlaminated chamber of an expansion machine of a rotational type in relation to the angle of turn of the rotor. The laminae of the machine can be placed radially or in an inclined plane. The profile of the transverse section of the cylinder has the form of two semicircles with displaced centers. V. D. Sokolov

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5156. Gaigerov, V. I., Influence of the adiabate indicator on the characteristics of a centrifugal compressor and axial gas turbine** (in Russian), Avtoref. Diss. Kand. Tekhn. Nauk, Mosk. Vyssh. Tekhn. Uch-shche, Moscow, 1957; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 2858.

**5157. Eremenko, A. S., and Saikovskii, M. I., Problem of computation of the aerodynamic qualities of the nozzles of turbomachines** (in Russian), *Sb. Trudī In-ta Teploenergetika Akad. Nauk USSR* no. 13, 99–103, 1956; *Ref. Zh. Mekh.* no. 4, 1958, Rev. 3972.

In order to evaluate the aerodynamic qualities of different arrangements of nozzles of turbomachines, authors propose to utilize not the coefficients of hydraulic losses but the coefficients of resistance $\zeta_r$ or loss $\zeta_n$, which are referred to the dynamic pressure at the inlet which is computed by the resulting velocities

$$\zeta_r = 1 + \frac{2\lambda_2 - \chi_2}{n^2\chi_1} - \frac{2\lambda_1}{n\chi_1}, \quad \zeta_n = 1 + \frac{2\lambda_2}{n^2\chi_1} - \frac{2\lambda_1}{n\chi_1}$$

Here $\lambda_{1,2}$ and $\chi_{1,2}$ are the coefficients of nonuniformity of the quantity of motion and kinetic energy at the inlet and at the outlet of the nozzle, $n$ is the degree of expansion of the nozzle.

V. I. Vasil'ev

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5158. Zhumakhov, I. M., Methods for improving fixed pumps and ventilators of the principal air ventilation systems of the mining industry** (in Russian), *Gorn. Zh.* no. 2, 39–45, 1956; *Ref. Zh. Mekh.* no. 4, 1958, Rev. 3993.

Author discusses the manufacture of pumps of high economic qualities and of ventilators for the mining industry, matching up to modern technical standards. The basic advantages of centrifugal spiral pumps over sectional pumps are indicated. In the field of mine-shaft ventilation equipment, author is of the opinion that axial ventilators with simple and economical controls of the performance of their work, effected by the turning of the wheel vanes and of the directing (guiding) apparatus, fulfil to the largest extent the requirements of a progressive regime of a mine-shaft ventilation system.

V. N. Gusev

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5159. Philippoff, W., Viscosity measurements on polymer modified oils,** *ASLE Trans.* **1,** 1, 82–86, Apr. 1958.

**5160. Pindzola, M., and Mozer, G. R., Investigation of a multiple-source schlieren system for application to a perforated wall wind tunnel** (in English), 9th Congrès Intern. Mécan. Appl., Univ. Bruxelles, 1957; **4,** 387–406.

The use of a multiple-source schlieren system was studied for application to wind tunnels employing perforated walls. Visibility equations are derived showing the influence of geometrical parameters on the schlieren photograph background pattern for first one and then two tunnel walls. A uniformly illuminated background pattern is shown to be easily attained with a single perforated wall, but not with two identically perforated walls. A method of obtaining a uniformly illuminated background pattern for the two-wall case by relocating the holes in one perforated tunnel wall is discussed. The use of an external source plate is shown to increase system sensitivity at no sacrifice in background pattern. The multiple-source schlieren system is also shown to be useful with transparent walls where the conventional system is unsatisfactory.

From authors' summary

**5161. Shepard, C. E., and Boldman, D. R., Preliminary development of electrodes for an electric-arc wind tunnel,** *NASA Memo* 4-14-59E, 15 pp., Mar. 1959.

Two electrode configurations were tested in an electric-arc wind tunnel at the NASA Lewis Research Center. The results indicated approximately the same heat-loss rate per unit of arc power input for each of the configurations. Measured heat-loss rates were on the order of 40% of the arc power input. Nearly all this loss occurred at the anode. The power input and arc current limitations of the electrodes appear to be the critical design factors.

From authors' summary

**5162. Weske, J. R., Application of the electric spark method to the investigation of transonic flow past airfoils and airfoil grids,** AFOSR TN-58-304 (ASTIA AD 154-214; Inst. Fluid Dynam. Appl. Math., Univ. Maryland, TN BN-128), 14 pp., Apr. 1958.

**5163. Squire, W., Hertzberg, A., and Smith, W. E., Real gas effects in a hypersonic shock tunnel,** Cornell Aero. Lab. Rep. AD-789-A-1, 72 pp., Mar. 1955.

Report discusses the effect of real-gas properties of air on different shock-wave variables. Its principal chapters are: Speed of sound in a real-gas; Normal shock calculation in a real gas; Experimental studies of the flow behind normal shock waves; Oblique shock waves in a real-gas; Isentropic flow calculation in a real-gas (Riemann characteristics, Prandtl-Meyer flow, Nozzle design); Hypersonic shock-tunnel design calculation.

Author's calculations (1000 K $\rightarrow$ 8000 K; $10^{-5} \rightarrow 10^{2}$ atmospheres) show that pressure and enthalpy behind normal shock waves are not appreciably changed from ideal-gas values, but density, temperature and flow velocities are significantly altered. In the case of the hypersonic shock tunnel (developed at Cornell Aeronautical Laboratory since 1951) the entrance Mach number (for constant driver shock conditions) is increased with respect to the ideal-gas value, causing a change in the nozzle configuration required to obtain a given test section Mach number. Overall expansion ratio for a given test section Mach number is increased; test section density, Reynolds number and testing time are reduced. A further consequence of the real-gas properties is an altering of the attached wave angles and wedges (at constant flow conditions) to somewhat smaller values than corresponding ideal-gas values; this fact will appreciably change Mach number determination.

Authors based their calculations on thermodynamic data, assuming a value of 7.37 *eV* for the dissociation energy of nitrogen rather than the now generally accepted value of 9.76 *eV*. The error introduced by using the smaller value is in the order of 5% for the ratio shock velocity: speed of sound behind a shock in pure nitrogen. Report shows clearly the importance of considering real-gas effects in both tunnel design and flow about test bodies.

C. Franze, Germany

**5164. Champion, P. A., Some factors involved in the supersonic wind tunnel testing of models,** *J. Roy. Aero. Soc.* **62,** 576, 893–897, Dec. 1958.

**5165. Krenzel, N. R., Discussion of photographic systems for recording shock-wave formations produced by supersonic sleds,** *J. Soc. Mot. Pict. Telev. Engrs.* **68,** 3, 147–148, Mar. 1959.

Four photographic systems—the shadowgraph, Toepler's schlieren, ballistic synchro, and the interferometer—are available for recording shock waves produced by aerodynamic configurations on track sleds traveling at supersonic velocities. A study has been undertaken at the Supersonic Naval Ordnance Research Track, China Lake, to determine which of these techniques are applicable to track-sled testing programs. A brief description of each system is given along with some preliminary test results obtained from the ballistic synchro system.

From author's summary

**5166. Dumitrescu, L., A shock tube for aerodynamic research** (in Roumanian), *Studii Si Cercetari Mecan. Appl.* **10,** 1, 271–285, 1959.

Paper presents a shock tube designed and constructed at the Institute for Applied Mechanics Traian Vuia in Bucharest. The tube is intended to replace conventional wind tunnels for high-speed aerodynamic research. The main characteristics of the

equipment and the auxiliary installations are described. Length of the tube is 10.600 m, while working section is 300 × 190 mm. The vacuum pump establishes a vacuum of 2 mm Hg abs, the maximum pressure ratio being 2000.

Results of preliminary technical tests and some flow visualizations at Mach number 2.4 are also given.

A. Petre, Roumania

**5167. Vasil'ev, L. A., Semenov, S. S., and Tarantov, E. A., Study of physical processes in a shock tube with the aid of high-speed photography** (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 11, 186–188, 1957.

A high-speed shadowgraph for use in shock tubes is described. It has a maximal frequency of 100,000 pictures per second and a maximum capacity of 7600 shots of 3.6 × 4.8-mm size. The shadowgraph device IAB-451 has a collimator part before the measuring chamber and a receiving part behind it, which are composed equally of a concave mirror and a meniscus lense. The shadowgraph is used in connection with the high-speed camera Type FP-22, a description of which has been given by L. A. Samirov [*Zh. Nauch. Prikl. Fotografii i Kinemat., Akad. Nauk SSSR* **1**, no. 4, 1956]. The light intensity of each shot in dependence on time has the form of a roof, and a formula is derived for the real exposure time dependent upon the film sensitivity and the total exposure time. For a frequency of 100,000 pictures per second the total exposure time is 14.5 $\mu$s but for the real one only 7 $\mu$s. Series of pictures are given for the interference of an incident shock wave on an inclined wedge.

W. Wuest, Germany

**5168. Hubbard, H. H., Noise problems associated with ground operations of jet aircraft,** *NASA Memo* 3-5-59L, 13 pp., Mar. 1959.

The nature of the noise-exposure problem for humans and the aircraft-structural-damage problem are each discussed briefly. Some discussion is directed toward available methods of minimizing the effects of noise on ground crews, on the aircraft structure, and on the surrounding community. A bibliography of available papers relating to noise-reduction devices is also included.

From author's summary

**5169. Lina, L. J., Champine, R. A., and Morris, G. J., Flight investigation of an automatic throttle control in landing approaches,** *NASA Memo* 2-19-59L, 8 pp., Mar. 1959.

The investigation showed that airspeed could be maintained satisfactorily by an automatic throttle control. Turbulent air caused undesirably large variations of engine power which were uncomfortable and disconcerting; nevertheless, the pilot felt that he could make approaches 5 knots slower with equal assurance when the automatic control was in operation.

From authors' summary

## Thermodynamics

(See also Revs. 4916, 5023, 5149)

**Book—5170. Spalding, D. B., and Cole, E. H., Engineering thermodynamics,** London, Edward Arnold, Inc., 1958, viii + 375 pp. 28s.

Book is designed to give first-year engineering students, regardless of later specialties, a grounding in thermodynamics. It is therefore confined to basic (mostly elementary) principles, with few applications except simple illustrative examples. Beginning with introductory survey and definition of units, it discusses, in order, work, temperature, heat, First Law, pure substances, steady-flow processes, properties of pure substances, Second Law, reversibility, absolute temperature scale, entropy, ideal gases, gaseous mixtures, and combustion.

Treatment leans heavily on "work" and "heat" definitions characterizing Keenan's *Thermodynamics.* Although this is an excellent approach, reader may gain impression of somewhat dogmatic attitude that this is the only proper approach. Perhaps a different choice of words or phrases would be less unsettling to first-year student who is told emphatically that such a common word as "heat", used and heard by him all his life, does not mean what he thought or dictionary says it means. How many beginning students are competent to say whether "sole" effect of an operation "could be reduced" to the rise of a weight?

There is considerable opinion that barriers between different branches of engineering should be broken down. Persons holding this view would probably feel that book should at least cover "aerothermodynamics." Many also believe it better not to put too much emphasis on exactness at very beginning of a study; after all, general concepts came first in development of thermodymanics, just as for most sciences, with rigidity of proofs and exactness following later. Some readers may take exception to other points, not because authors are wrong, but because matters really involve differences of opinion (e.g., discussion of Bernoulli's equation, definition of displacement work, relegation of ideal gases to chapter 14).

Reviewer offers no personal opinion with respect to foregoing, noting merely that such criticisms may be made. However, he believes there is somewhat too much definition and discussion of various units, which should be minor matters in a book devoted to basic principles. (Why do many authors feel it necessary to adopt some particular units, when physical relations can be expressed in general form suitable for use with any system of units preferred by reader?) However, merits of book, including its general clarity, far outweigh any criticisms based on personal opinion, and reviewer definitely recommends it as a worthy and useful addition to existing texts.

C. W. Smith, USA

**5171. Chambadal, P., Thermodynamics properties of gas mixtures** (in French), *Chaleur et Industrie* **39**, 401, 303–313, Dec. 1958.

**Book—5172. Grubenmann, M., I, X-Diagrams of humid air and their application to heating, cooling, humidifying, dehumidifying, cooling towers and drying (I, X-Diagramme Feuchter Luft—und ihr Gebrauch bei der Erwärmung, Abkuhlung, Befeuchtung, Entfeuchtung von Luft bei Wasserruckkuhlung und beim Trocknen),** 4th enlarged edition, Berlin, Springer-Verlag, 1958, 41 pp. DM 15.60 (Paperbound).

The four hygrometric charts could possibly be of use if they were accompanied by adequate explanations. The book of 41 pages is far removed from fulfilling this purpose. The early sections deal with the basic thermodynamics of hygrometry and the construction of charts, but the reader is left wondering whether the author knows the differences between heat and enthalpy (or caloric?), enthalpy and specific heat, nonflow and flow processes. A later section deals with the drying of nonhygroscopic materials, and the heat and mass transfers associated with it. This section is an abstract from a paper by E. Kirschbaum. The remaining sections deal with applications to particular systems, but the reader will gain little guidance from them. Thus one section deals with a particular type of drying tower, not described, and a series of design data are quoted without any information of how these have been obtained from the charts. The chief merit of the device appears to be that it was designed by E. Kirschbaum to whom the author seems to be very indebted, and he feels obliged to drag in references and profuse acknowledgements to his work as frequently as possible, to the detriment of sound explanations.

As far as is possible to judge, the charts themselves are not based on very recent thermodynamic data.

Y. R. Mayhew, England

**5173. Wakeshima, H., and Takata, K., On the limit of superheat,** *J. Phys. Soc. Japan* **13,** 11, 1398–1403, Nov. 1958.

In a column of liquid a temperature gradient of about 0.5C/mm was maintained. Drops of insoluble liquids (saturated hydrocarbons, polymethylenes, and ethyl ether) rising in this column were observed to explode when heated to a definite temperature, independent of drop size (0.02–0.6 mm). For these liquids the limit of superheat, defined as the difference between explosion temperature and boiling point, ranged from 110 to 135C. These values agree with Döring's theory of spontaneous bubble formation in superheated liquids.

H. A. Vreedenberg, Holland

**5174. Koeppe, W., On the temperature dependence of the specific heat of gases at constant pressure** (in German), *Forsch. Geb. Ing.-Wes.* **24,** 5, 161–164, 1958.

Using the equation of state and the integral Thomson-Joule effect it can be shown that, for each gas, a lower and an upper limiting temperature exist at which the specific heat at constant pressure reaches its value for zero pressure at the pressure under consideration. In the region between these two limiting temperatures the specific heat, at the given pressure, exceeds its value for zero pressure. From author's summary

**5175. Nachbar, W., Williams, F., and Penner, S. S., The conservation equations for independent coexistent continua and for multicomponent reacting gas mixtures,** *Quart. Appl. Math.* **17,** 1, 43–54, Apr. 1959.

Authors summarize as follows: "The equations for conservation of mass, momentum, and energy are derived for a set of independent, coexistent continua. A direct comparison is conducted between the continuum-theory results and those obtained from kinetic theory by assuming that, for each of the species, the kinetic theory definitions apply. It is found that the new terms appearing in the conservation equations derived from continuum theory are precisely those which are required to make these equations identical with the results obtained from the kinetic theory of multicomponent, reacting gas mixtures."

Reviewer feels that the article is a welcome addition to the literature on fundamental principles. The condition of independent components may be relaxed by using the concept of partial quantities; for example partial enthalpy instead of authors' total average specific enthalpy. L. S. Dzung, Switzerland

**5176. Pope, A. W., Design and development of four lightweight high-speed marine gas turbines for electric generator drive,** *Proc. Instn. Mech. Engrs.* **172,** 8, 301–319, 1958.

**5177. Woods, W. A., On the importance of engine pressure level to turbo-jet engine reheat performance,** *J. Roy. Aero. Soc.* **63,** 580, 263–265 (Tech. Notes), Apr. 1959.

**5178. May, C. E., Koneval, D., and Fryburg, G. C., Stability of ceramics in hydrogen between 4000° and 4500°F,** *NASA Memo* 3-5-59E, 13 pp., Mar. 1959.

The extent of reaction of hydrogen with various carbides, nitrides, borides, and oxides was studied in a static system. Most of these materials, HfC, ZrC, TiC, TaC, NbC, WC, $Mo_2C$, HfN, ZrN, NbN, $ZrB_2$, $NbB_2$, and WB, showed less reaction than the minimum detectable value. However, the data seem to indicate that the ceramics, TiN, TaN, $HfB_2$, $TiB_2$, $ZrO_2$, and $Cr_2O_3$, had reacted. Reactions of SiC, VC, and $TaB_2$ were not determinable because of their incompatibility with the tungsten container.

From authors' summary

**5179. Martin, W. H., and Rixon, J., Apparatus for the measurement of specific heat of metals,** *J. Sci. Instrum.* **36,** 4, 179–182, Apr. 1959.

An apparatus is described which enables the measurement of the specific heat of metals to be made *in vacuo* up to a temperature of at leat 500C.

The specimen in the form of a flat cylinder has its temperature steadily increased by a metered amount of electrical energy supplied by an internal heater. Radiative heat losses are eliminated by means of an adiabatic screen which surrounds the specimen and which is maintained at the same temperature. The rate of temperature rise can be controlled by the energy input, enabling specific heat to be investigated over small temperature ranges. An accuracy of within ±3% has been obtained at temperature up to 500C. From authors' summary

**Book—5180. Elliott, J. F., edited by, The physical chemistry of steel making** (from Proc. Conference on Physical Chemistry of Iron and Steelmaking, 1956, sponsored by Dept. of Metallurgy, (M.I.T.), New York, John Wiley & Sons, Inc., 1958, xiii + 257 pp. $15.

Book contains the 42 papers and discussion summaries of the conference. It is divided into the following sections: 1. Liquid metals and properties in liquid iron and steel; 2. Equilibria of reactions in liquid iron and steel; 3. Behaviour of metal oxides and of components of iron and steelmaking slags; 4. Slag-metal equilibria in blast-furnance and steelmaking furnace systems; 5. Kinetics and slag-metal reactions; 6. Reaction rates in iron and steelmaking processes; 7. Application of fundamental data to process development and metallurgical problems in the steel industry; 8. Solidification of castings and ingots; 9. Research planning; 10. Special lecture on kinetic problems in steelmaking, by C. Wagner.

The aim of the conference (to bring together metallurgists who were actively working in the field and who were qualified by background and experience to contribute significantly, and to obtain a survey on the latest research and a better understanding of actual steel-making systems) is fully reached. The fact that the physical chemistry of metals involves many differences from the "classical chemistry," which makes necessary a special kind of thinking, moves throughout the book. It seems that in the investigation of the complicated slag-metal reactions and equilibria considerable progress has been made, and it becomes evident that fundamental results obtain an increasing significance for practical metallurgy. Thus book presents a good survey on the basic investigations as well as their applications to steelmaking in America and Europe. It is to be hoped that this conference will not be the last one of this kind.

Publishers have done an excellent job, and price is reasonable, but such a long time between conference and publication should be avoided in future. A. Kochendörfer, Germany

**5181. Uchiyama, S., Theory of thermoelectric refrigeration,** *Mem. Fac. Engng., Univ. Nagoya* **9,** 2, 321–329 (Research Reports), Nov. 1957.

Paper extends Altenkirch's theory for temperature difference in thermoelectric refrigeration to include the Thomson effect. Author claims that his modification yields lesser maximum temperature differences and greater optimum currents than those predicted by Altenkirch theory and that his theoretical results agree better with experiment. In addition, the possibility of increasing maximum temperature differences by changing either the resistance or the cross-sectional area of semiconducting elements along the current path is investigated. Author concludes that changing resistance is the more effective of the two possibilities, yielding predicted increases in maximum temperature difference of as much as 14% for a Bi-Te alloy.

R. L. Young, USA

**5182. Howells, H. R., Marine refrigeration** (The 31st Thomas Lowe Gray Lecture), Instn. Mech. Engrs. Prepr., 13 pp., 1959.

**5183. Hatsopoulos, G. N., and Keenan, J. H., Analysis of the thermoelectric effects by methods of irreversible thermodynamics,** *J. Appl. Mech.* **25,** 4, 428–432, Dec. 1958.

By means of the thermodynamics of irreversible processes the thermoelectric coefficients are expressed in terms of a temperature function $h$. The connection is that $h$ is the difference between the heat of transfer and the potential. As far as the reviewer can see there is no essential difference from the treatment given by S. R. de Groot ["Thermodynamics of irreversible processes," Amsterdam 1951], whose work is not included among the references. L. J. F. Broer, Holland

**5184. Aronson, D., Preliminary design studies of low temperature refrigeration plants. Parts I and II,** *Chem. Engng. Progr.* **54,** 11, 77–80, Nov. 1958; **54,** 12, 76–80, Dec. 1958.

**5185. Harman, T. C., Special techniques for measurement of thermoelectric properties,** *J. Appl. Phys.* **29,** 9, 1373–1374 (Letters to the Editor), Sept. 1958.

**5186. Crabol, J., and Van Kote, J., Infrared pyrometer for the temperature measurement of blades in turbines** (in French), *Rech. Aéro.* no. 66, 3–11, Sept./Oct. 1958.

## Heat and Mass Transfer

(See also Revs. 5107, 5147, 5262, 5293)

**Book—5187. Kutateladze, S. S., Basis of the theory of heat exchange** (in Russian), Moscow-Leningrad, Mashgiz, 1957, 385 pp. + illus. 12r 65k; *Ref. Zh. Mekh.* no. 4, 1958, Rev. 4276.

An exposition is given of the basic principles of heat transfer in solid bodies, heat transfer in cases of constrained and free convection, heat transfer during changes in the state of aggregation of the heat-carrier, radiation heat exchange and an analysis of the experimental data. The first five chapters describe the basic concepts relating to heat transfer, and equations for the motion of inflow of heat are brought in, both for the laminar as well as the turbulent conditions of flow. Boundary and initial conditions are discussed and the conditions of the mechanical and thermal interaction on the boundaries of phase divisions in multiphase systems. Chapter VI gives a concept of the similarity of physical phenomena and, in particular, conditions are deduced for the similarity of heat-exchange processes. In the next two chapters solutions are examined of stationary and nonstationary problems of heat conductivity in solid bodies, in their simplest form, principally one-dimensional. Chapter IX is devoted to investigations of questions relating to heat transfer in laminar and turbulent flows of an incompressible liquid in cylindrical tubes, and to the influence of heat transfer on the velocity profile, and heat transfer in the transition region. In chapters X and XI questions are answered in regard to heat transfer during extemal flow past bodies by an incompressible liquid and by a gas at high flow velocities. Experimental data are furnished regarding the turbulent heat transfer from a plate, a sphere, a cylinder, from an assembly of pipes in a transverse flow and heat transfer when gas is flowing in a tube at high velocities. Elementary information is given in chapter XII on friction and heat exchange in rarefied gases, Chapter XIII is devoted to an exposition of the exact solution of the problem of free convection near the vertical wall. The experimental results for turbulent free convection are reproduced. The next two chapters deal with questions relating to heat transfer during steam condensation on solid surfaces and condensation on a free stream of liquid. In chapters XVI and XVII an investigation is carried out regarding heat transfer when liquids boil and regarding the nature of boiling in liquids. Chapter XVIII is given up to the exposition of the basic principles of radiation heat transfer of solid bodies and gases and some special features of radiation in flaming gases, flames and radiation in furnaces. In the last chapter, XIX , reference is made to some cases of complex heat exchange. In the supplement a summary is furnished of the basic calculation formulas on heat transfer, with tables and graphs of the physical properties of the very diverse materials concerned. The book is written for engineer - heat specialists and could be used by students intending to specialize in heat technics.

G. A. Tirskii
*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**Book—5188. Rykalin, N. N., Calculation of the heat transfer processes in welding [Berechnung der Wärmevorgange beim Schweissen],** Berlin, VEB Verlag Technik, 1957, 326 pp. DM 38.

This translation of the Russian original is designed as a reference for the welding engineer and a text for the student of mechanical engineering.

The first chapter gives an account of fundamentals of transient heat conduction at a rather elementary level. Numerous solutions of problems involving concentrated and distributed heat sources are worked out in detail, assuming Laplace's solution. A very sketchy treatment of numerical solutions for one- and two-dimensional problems is given. The problem of penetration of melting boundary is not discussed.

The second and third chapters describe the heating up of metals in arc and oxyacetylene welding, respectively. Approximate solutions for moving heat sources are derived for typical idealized configurations. The quantitative treatment of thermal losses from the arc and flame is based entirely on tests of standardized Russian equipment and the only gas for which data are quoted is acetylene. The numerous assumptions, however, leading to the simplified calculation schemes are discussed in detail.

Chapter four is devoted to the calculations of the melting process in electrodes and the weld during arc welding. The problems are simplified to a stage in which essentially heat-balance-type calculations are employed.

The last chapter describes the effects of temperature and of the heating and cooling rates on the properties of the weld. Methods of calculation of these rates are included and methods of control are suggested.

The reviewer believes the book to be of limited value to the practicing engineer in this country since the theoretical methods employed are fully covered in standard texts [Carslaw, H. S., and J. C. Jaeger: "Conduction of heat in solids," 2nd ed., London, Oxford Univ. Press, 1959; Jakob, M.: "Heat transfer," v. 1, New York, John Wiley and Sons, 1949] while the data referring to the complex thermal processes in the arc or flame are restricted to standard Russian equipment and extrapolation does not appear feasible. On the other hand the text presents a comprehensive treatment of welding processes from the point of view of heat transfer and as such has no counterpart in English literature.

The bibliography contains 34 references, exclusively Russian.

J. R. Moszynski, USA

**5189. McFarland, B. L., Comparison between the linear and nonlinear steady-state behavior of a heated tube,** *J. Appl. Phys.* **29,** 12, 1682–1684, Dec. 1958.

Paper provides an assessment of the errors involved in assuming constant thermal and electrical conductivity of heated tubes, used in measurements of forced-convection heat-transfer coefficients. The tube is assumed to be insulated externally. The steady-state heat-conduction equation is linearized by a modified Kirchhoff's transformation with two parameters. The parameters are selected to give the best approximation simultaneously to the temperature-dependent conductivity and rate of heat generation.

The resulting Bessel equation of order zero is further simplified by assuming a constant coefficient of the first derivative term.

A solution in terms of hyperbolic functions is obtained and its series expansion is compared with the solution of the linear problem. In the range of variables considered, the error due to assuming constant conductivities is found to be of the order of 1.5%, provided that the thermal conductivity is evaluated at the mean tube temperature, while the electrical conductivity is evaluated at the outer surface temperature.

Author's notation is somewhat confusing in that some symbols are not defined and hyperbolic functions are identified with trigonometric functions.

Reviewer doubts the value of the final simplification of the Bessel equation since it appears to introduce errors of the same order of magnitude as the nonlinear effects.

J. R. Moszynski, USA

**5190. Begunkova, A. F., The effect of contact thermal resistances of a pellet insulation** (in Russian), *Teploenergetika* no. 12, 85–86, Dec. 1958.

Data are presented concerning the properties of a pellet insulation, as well as the results of tests of the deformed pellets.

From author's summary

**5191. Portman, D. J., Conductivity and length relationships in heat-flow transducer performance,** *Trans. Amer. Geophys. Un.* **39,** 6, 1089–1094, Dec. 1958.

In recent years many investigators have used thermoelectric heat-flow transducers to measure heat conduction in natural soil. If the transducer has thermal properties different from those of the soil it will interfere with the natural heat transfer and an error in measurement will occur. An analysis of the error for a transducer placed in an infinite medium in steady-state conditions is made in terms of two nondimensional parameters: (1) the ratio of transducer conductivity to soil conductivity, and (2) the ratio of transducer thickness to transducer width. An exponential relationship is postulated on the basis of physical considerations and is verified by the results of an electrical analogy experiment for a square transducer having a thickness-width ratio of 0.125 for conductivity ratios from about 0.1 to 2.2. An error of −70% was found for a conductivity ratio of 0.1 and an error of +5% for a conductivity ratio of 2.2. The results are used to estimate error in measurements made during the Great Plains Turbulence Field Program.

From author's summary

**5192. Beck, A. E., and Beck, Julia M., On the measurement of the thermal conductivities of rocks by observations on a divided bar apparatus,** *Trans. Amer. Geophys. Un.* **39,** 6, 1111–1123, Dec. 1958.

Rapid techniques for the preparation of disks of rock and for the determination of rock conductivities have been used to make a large number of measurements on both coarse-grained and fine-grained rocks. Methods in which either sets of disks or single disks can be used are described, and the mean values of conductivity found by combining the results in various ways are compared. The modes of the rocks were found by examination of thin sections for the fine-grained rocks and by examination of each disk for the coarse-grained rocks. Conductivities computed from the modes by the method of Birch and Clark are compared with the experimentally determined conductivities. The significance of rock structure in the selection of representative specimens and the effect of moisture content in porous rocks is discussed. The work indicates that, at least for the coarse-grained rocks, the usefulness of the conductivities obtained from divided bar measurements can be seriously questioned, and that the usual methods of saturating porous rocks are not wholly satisfactory.

From authors' summary

**5193. Duffin, R. J., A variational problem relating to cooling fins,** *J. Math. Mech.* **8,** 1, 47–56, Jan. 1959.

A mathematical solution is given for the taper of a cooling fin which will provide maximum heat dissipation for a given volume. A rigorous proof is given that the taper should be such as to produce a linear variation of temperature from the root of the fin to its tip. The taper is then calculated for a rectangular and a circular fin so as to satisfy the condition of uniform temperature gradient. A method is also given for estimating the loss in cooling efficiency incurred by the use of a non-optimum design.

The solution of the cooling fin problem has been known since 1926. This paper, however, contributes a rigorous mathematical justification of this solution, plus an estimate of the penalty of off-optimum designs. While neat mathematically, from an engineering point of view the result is an exact mathematical treatment of an equation which only approximately represents the heat dissipation.

L. H. Schindel, USA

**5194. Gvozdev, A. A., Settlement stresses, due to temperature, on concrete blocks and massive installations** (in Russian), *Sbornik Trudi Mosk. Inzh.-Stroit. In-ta* no. 17, 11–43, 1957; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 3411.

Graphic and analytical methods are put forward for the approximate solution of the problem of the propagation of heat in concrete blocks and massive installations. Then, formulas are deduced for the calculation of the thermal and settling deformations and stresses, with considerations for the effect of creep. The deduced expressions are applied to the concrete blocks of hydrotechnical installations, using different types of fastening for the blocks. The calculations of the thermal and shrinkage stresses are carried out with the view to determine the maximum permissible dimensions for the blocks. In order to increase these dimensions a number of proposals are made, namely: reducing the quantity of cement used in the mixture, using cement with low heat emission, cooling of the concrete by means of a system of pipes incorporated in the concrete. However, the last suggestion is costly. Of more practical value would be the cooling of the concrete before use, if the concrete-setting is done at the hot time of the year. In suitable cases it is expedient to cover the concrete surface with some heat insulating material and to adopt a filler with a low coefficient of thermal expansion.

K. S. Zavriev

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5195. Mills, J., An analogue solution of the heating of a power cable buried in the soil,** *J. Instn. Engrs. Austral.* **30,** 3, 91–98, Mar. 1958.

Paper describes solution of problem of heating of a power cable buried in the soil by combining the analogies of field plotting and use of an electric network. A ladder network of resistors and capacitors, with d.c. current input, and with associated electrical equipment enabling measurement of the potential differences which parallel the temperature differences in practice, provides a simple analog computer enabling rapid determination of the transient cable temperatures as a function of time. Field plotting techniques, as described in the Appendix, afford information needed in setting the resistances and capactors of the computer.

Comparison of analog-obtained values with calculated values for several known cases, for both steady and cyclic loading of the cables, evidences the good accuracy afforded by the analog. Finally, comparison of analog values and of experimental values obtained on an actual laboratory-scaled physical system both evidences good agreement and indicates the actuality of usefulness in practice of this relatively inexpensive special analog computer.

T. J. Higgins, USA

**5196. Reznikov, A. N., The temperature field produced in a chip under the action of frictional forces on the front face of a tool,** *J. Tech. Phys.-Soviet Phys.* **3,** 1, 182–192, Aug. 1958. (Translation of *Zh. Tekh. Fiz., Akad. Nauk SSSR* **28,** 1, 207–217, Jan. 1958 by Amer. Inst. Phys., Inc., New York, N. Y.)

Temperature distribution in a two-dimensional chip is studied analytically by use of the concept of moving sources. Heat flow due to friction between chip and front face of tool is considered, and the possibility of superposing the effect of heat generated in material deformation is mentioned but not studied. Chip is taken to be adiabatic except for frictional heat supplied. Solutions are developed both with and without longitudinal conduction in chip, and two temperature contour diagrams are presented.

J. E. Plapp, USA

**5197. Vodicka, V., A contribution to the unit representation of temperature functions in the case of immovable sources of heat** (in English), *Arch. Mech. Stos.* **10,** 6, 783–792, 1958.

The function determining the distribution of the heat sources in the heat equation is a sum of Heaviside functions. The presence of instantaneous and continuous heat sources is assumed, acting at various moments $t_i$ and various time intervals $(t_k, t_l)$, $t > 0$. Using the property of the Heaviside function, author derives a formula giving a uniform expression of the temperature function in an infinite isotropic medium at any time $t > 0$. The function considered is a sum of functions for instantaneous and continuous sources as a result of superposition for the linear heat equation.

J. Ignaczak, Poland

**5198. Mager, A., Supersonic airfoil performance with small heat addition,** *J. Aero/Space Sci.* **26,** 2, 99–107, Feb. 1959.

The addition of heat to flow near airfoils alters the pressure distribution and influences the drag and lift coefficients. Investigation is necessary to determine if improved aircraft performance is possible.

A linearized analysis for supersonic airfoils in shockless, inviscid flow is made based on the results of Tsien and Beilock's analysis [*J. Aero. Sci.* **16,** Dec. 1949; AMR **3** (1950), Rev. 2002] of the effect of a linearized heat source on uniform supersonic flow. The aerodynamic performance is compared for a similar airfoil and conditions with those computed by Pinkel, Serafini, and Gregg [*NACA RM* E51K26, Feb. 1952] using method of characteristics. Agreement is satisfactory for small heat additions for pressure distributions and for lift and drag coefficients.

Calculations are made to compare total energy requirements (propulsion plus heating) for several illustrative cases. The advantage of airfoil head addition is shown to be marginal and, for the most practical of the illustrative cases, is nonexistent.

R. H. Eustis, USA

**5199. Friend, W. L., and Metzner, A. B., Turbulent heat transfer inside tubes and the analogy among heat, mass and momentum transfer,** *AIChE J.* **4,** 4, 393–402, Dec. 1958.

Turbulent flow inside tubes has been studied for a wide range of fluid physical properties of several Newtonian liquids, with a variation in Prandtl number from 50 to 600.

Heat-transfer rates were measured with moderate temperature differences, and it is shown that the generally accepted semi-theoretical analogies between heat and momentum transfer are not representative at high Prandtl numbers. A final correlation formula is presented which represents all the available data with standard deviation of 9.4%, for low and moderate Schmidt number.

E. A. Brun, France

**5200. Scott, C. J., The application of constant property solutions to mass transfer cooling calculations,** AFOSR TN 59-201 (Univ. Minn., Inst. of Technol., Rosemount Aero. Lab. Engng. Mem. 76; ASTIA AD 211 527), 32 pp., Dec. 1958.

Simple transformation relations are developed for applying the results of constant-property boundary-layer solutions to the calculation of mass-transfer cooling problems using foreign gas effusion. The methods are similar to those developed by Eckert for solid surface predictions. Laminar constant pressure, laminar stagnation, and turbulent constant pressure flows are studied. A reference concentration of four-tenths of the wall concentration was used in evaluating the properties. Comparisons are made with currently available exact theoretical solutions in order to assess the value of the method as a means for extrapolating to conditions not covered by the available solutions. The important criterion for obtaining close agreement is that of wall concentration. If the wall concentrations are estimated accurately then good correlation results between present calculations and exact analysis.

From author's summary

**5201. Reynolds, W. C., Kays, W. M., and Kline, S. J., Heat transfer in the turbulent incompressible boundary layer, Part I. Constant wall temperature,** *NASA Memo* 12-1-58W, 48 pp. + 12 figs., Dec. 1958.

Heat-transfer rates, velocity profiles, and temperature profiles for the turbulent incompressible flow of air over a flat plate with a constant surface temperature have been measured at Reynolds numbers up to $3.5 \times 10^6$. The turbulent heat-transfer measurements agree well with the von Karman analogy, and the velocity profiles agree with the data of previous investigators. The temperature profiles are similar to the velocity profiles, both being adequately described by power formulas.

From authors' summary by S. Corrsin, USA

**5202. Reynolds, W. C., Kays, W. M., and Kline, S. J., Heat transfer in the turbulent incompressible boundary layer, Part II. Step wall-temperature distribution,** *NASA Memo* 12-2-58W, 25 pp., + 2 tables + 9 figs., Dec. 1958.

Heat-transfer rates and temperature profiles for the turbulent incompressible flow of air over a flat plate with a stepwise temperature distribution (unheated starting length) were measured for a variety of step positions at Reynolds numbers up to $3.5 \times 10^6$. Comparison of the data with existing heat-transfer analyses indicates that an improved analysis is needed. An integral analysis is made that agrees very well with the data and allows a simple correction for the unheated starting length. In addition, a differential analysis is made that allows prediction of the temperature profiles from the velocity profiles, and good agreement with experimental profiles is obtained.

Reviewer notes that no comparison is made with the data of D. S. Johnson [*J. Appl. Mech.* **24,** no. 1, Mar. 1957].

From authors' summary by S. Corrsin, USA

**5203. Reynolds, W. C., Kays, W. M., and Kline, S. J., Heat transfer in the turbulent incompressible boundary layer. Part III. Arbitrary wall temperature and heat flux,** *NASA Memo* 12-3-58W, 35 pp. + 5 tables + 12 figs., Dec. 1958.

Superposition techniques are used to calculate the rate of heat transfer from a flat plate to a turbulent incompressible boundary layer for several cases of variable surface temperature. The predictions of a number of these calculations are compared with experimental heat-transfer rates, and good agreement is obtained. A simple computing procedure for determining the heat-transfer rates from surfaces with arbitrary wall-temperature distributions is presented and illustrated by two examples. The inverse problem of determining the temperature distribution from an arbitrarily prescribed heat flux is also treated, both experimentally and analytically.

From authors' summary by S. Corrsin, USA

**5204. Reynolds, W. C., Kays, W. M., and Kline, S. J., Heat transfer in the turbulent incompressible boundary layer. Part IV. Effect of location of transition and prediction of heat transfer in a**

**known transition region,** *NASA Memo* 12-4-58W, 20 pp. + 1 table + 7 figs., Dec. 1958.

Using some of the turbulent heat-transfer results obtained in the foregoing papers [Revs. 5201, 5202, 5203], authors compute heat transfer for a transition-region model which is created somewhat in the spirit of Emmons [*J. Aero. Sci.* **18,** no. 7, July 1951; AMR **5** (1952), Rev. 486]. The principal difference is that this model is stationary, with an irregular line of transition rather than a random distribution of turbulent "spots." S. Corrsin, USA

**5205. Dusinberre, G. M., Effective and local surface coefficients in fin systems,** *Trans. Amer. Soc. Mech. Engrs.* **80,** 7, 1596–1597 (Tech. Briefs), Oct. 1958.

When tests are made on systems containing extended heat-transfer surface, the direct result is an "effective" surface coefficient. The desired result is a "local" or true surface coefficient. This paper shows how, on reasonable assumptions, the one can be calculated from the other without trial and error procedures.

From author's summary

**5206. Sibulkin, M., Estimation of turbulent heat transfer at the sonic point of a blunt-nosed body,** *Jet Propulsion* **28,** 8, 548–554, Aug. 1958.

An extension of the author's previous paper [*J. Aero. Sci.* **23,** no. 2, Feb. 1956; AMR **9** (1956), Rev. 3393] yields expressions for the heat transfer at the sonic point of a blunt body. Results for flow conditions at the sonic point for perfect and imperfect gases and heat transfer are given in terms of conditions at the stagnation point. Boundary-layer analysis is based on Blasius pipe-flow formula and the 1/7 power law for velocity and enthalpy. Additional approximations yield a very approximate formula for heat transfer. Comparison with the more detailed calculation is good. Sample calculations with perfect and real gas properties are given.

A. M. Kuethe, USA

**5207. Sparrow, E. M., and Siegel, R., Thermal entrance region of a circular tube under transient heating conditions,** Proc. Third U. S. Nat. Congr. Appl. Mech., June 1958; Amer. Soc. Mech. Engrs., 1958, 817–826.

An analysis is made for laminar forced convection heat transfer in the thermal entrance region of a circular tube whose wall may take on arbitrary time variations of either surface temperature or surface heat flux. The time varying process may begin either from an already established steady-state heat-transfer situation, or from a situation where wall and fluid are at the same uniform temperature. The velocity profile is taken to be steady and fully developed (Poiseuille flow). The first step in the analysis determines the thermal response to a unit step change in wall temperature or wall heat flux by using the method of characteristics. Then, by a superposition technique, this solution is generalized to apply for arbitrary time variations. The final results are expressed in the form of integrals which may be easily evaluated for particular applications. From authors' summary

**5208. Fastovsky, V. G., and Petrovsky, J. V., Heat transfer and the resistance to air flow in a bundle of sheets with semispherical projections** (in Russian), *Teploenergetika* no. 1, 65–68, Jan. 1959.

Formulas derived from experiments on models are presented for calculation of heat exchange and the resistance to flow. Practical applications of heat exchangers made of sheets with semispherical projections are considered. From authors' summary

**5209. Shitzman, M. E., Heat transfer to water, oxygen and carbon dioxide in the near critical region** (in Russian), *Teploenergetika* no. 1, 68–71, Jan. 1959.

A comparison is given between an earlier method of heat-transfer calculation in the near critical region and the new data derived from the experiments with water, liquid oxygen and carbon dioxide.

From author's summary

**5210. Omatsky, A. P., and Scherbakov, V. K., Heat exchange intensification in the critical region with the aid of ultrasound** (in Russian), *Teploenergetika*, no. 1, p. 84, Jan. 1959.

Data are given on the effect of ultrasonic vibrations in the cooling liquid upon the first critical point of heat exchange for a large volume. From authors' summary

**5211. Maslen, S. H., On heat transfer in slip flow,** *J. Aero. Sci.* **25,** 6, 400–401 (Readers' Forum), June 1958.

Author develops formulas that relate temperature, shear, and Nusselt number at the wall as a function of the continuum values. These expressions are derived from a first-order solution for a laminar compressible boundary layer over a flat plate previously presented by the author [AMR **6** (1953), Rev. 1342]. The heat transfer to the wall is considered to have two parts. One is due to conduction and the other is due to the sliding friction. The slip effect does not influence either the shear or the heat transfer. The shear is increased over its continuum value by the external pressure gradient induced by the boundary layer which is not caused by the slip effect. The heat transfer is unchanged from the continuum value. The heat flow due to sliding friction exactly compensates for the reduced conduction to the wall.

J. L. McCabria, USA

**5212. Wooding, R. A., An experiment on free thermal convection of water in saturated permeable material,** *J. Fluid Mech.* **3,** 6, 582–600, Mar. 1958.

Paper describes a fine combination of theory and experiment to discover the behavior of parameters governing the process of title. It is assumed that Darcy's law is valid for the liquid motion, that the packed sand behaves as a homogeneous permeable medium and that thermal conductivity is not affected by fluid dispersion. Differential equations for energy and motion are set up for the case of a space confined between two co-axial cylinders. Equations are solved by perturbation methods. Unknown coefficients are then derived from three experiments. The assumptions are in agreement with results of experiments.

J. H. van de Stadt, Holland

**5213. Tsubouchi, T., and Sato, S., On the heat transfer coefficients of cylindrical bodies, Part III** (in English), *Rep. Inst. High Speed Mech., Tohoku Univ. (B)* **9,** 81/90, 135–148, 1958.

In many previous investigations, the heat-transfer coefficients of the cylindrical surface free from the end effect or the mean heat-transfer coefficients over the whole surface of a cylindrical body have been measured, but coefficients of the individual surface affected by the other surface have hardly been studied. Authors measure the local and the mean heat-transfer coefficients of the vertical end surface and the horizontal cylindrical surface of circular cylinder, which is located in the still air horizontally, by means of "the temperature boundary-layer method" together with the usual "heat quantity method." The results of experiment are expressed in the experimental formulas on each part of cylinder and are compared with the conventional formulas of the vertical plates and the horizontal cylinders.

From authors' summary

**5214. Sidorov, E. A., Accounting for the effect of unstable conditions on convective heat exchange** (in Russian), *Teploenergetika* no. 4, 79–80, 1958.

In a constantly growing field of investigations about the effect of nonuniformity and time dependence of the main stream on the boundary-layer heat transfer, this paper makes a useful practical

contribution. It endeavors to derive a simple analytical expression to estimate the deviation in actual heat transfer from that when velocity and temperature variations of the main stream are neglected. The approach is based on the method of successive approximations in solution of the boundary-layer energy equation with proper boundary conditions imposed on the temperature field. Although more general analytical expressions for heat transfer have been obtained recently for either nonuniform temperature or velocity distribution of the main stream, this paper offers sufficiently simple and yet accurate method of estimating simultaneous effect of nonuniform temperature and velocity field of the main stream.

B. Zarwyn, USA

**5215. Singh, S. N., The determination of eigen-functions of a certain Sturm-Liouville equation and its application to problems of heat-transfer,** *Appl. Sci. Res.* (*A*) **7,** 5, 237–250, 1958.

A method for determining the discrete set of eigenvalues and the associated eigenfunctions of a certain Sturm-Liouville differential equation system is developed. The application of this approach to a number of physical problems is presented. In particular the problem of heat transfer by laminar flow in a tube with axial heat conduction, viscous dissipation and arbitrary heat sources and sinks is completed. L. Lapidus, USA

**5216. Singh, S. N., Heat transfer by laminar flow in a cylindrical tube,** *Appl. Sci. Res.* (*A*) **7,** 5, 325–340, 1958.

The problem of heat transfer of a fluid flowing through a tube in laminar flow is solved exactly, including the effects of axial heat-conduction, viscous dissipation and constant heat sources. For Peclet number > 100 the effect of axial conduction is shown to be negligible. Consideration is give to various types of wall temperatures. L. Lapidus, USA

**5217. Petoukhov, B. S., and Nolde, L. D., Heat exchange in viscous gravitational flow of liquid in pipes** (in Russian), *Teploenergetika* no. 1, 72–80, Jan. 1959.

An experimental investigation of heat exchange during the flow of liquid in a round pipe with different directions of forced and natural convection. Empirical formulas are offered for calculation of heat exchange under these conditions.

From authors' summary

**5218. Ostroumov, G. A., Free convection under the conditions of the internal problem,** *NACA TM* 1407, 189 pp. + 11 tables + 27 figs., Apr. 1958. [Transl. of Svobodnaya convectzis v ousloviakh vnoutrennei zadachi, Moscow-Leningrad, State Publ. House Technico-Theoretical Lit., 1952.]

Author considers linearized equations for natural convection of heat by gravity forces by assuming the ordinary boundary conditions at the wall and the conditions of a constant vertical temperature gradient (warmer below) and no vertical velocity gradient. A biharmonic differential equation is obtained and solved for the general case in cylindrical coordinates. Physically the model would be a hole drilled in block of solid matter and filled with fluid. The solution for steady convection in a vertical round channel is given for the special boundary conditions of a constant horizontal temperature gradient at infinity in the surrounding medium. Consideration is given to the effect of a wall between the free convection regime and the surrounding medium, the role of pumping the fluid, and also the case when the fluid is warmer above.

The unsteady-heat-transfer problem of a casting poured into a mold is attacked by dividing the process into three stages: initial mixing, development of convective pattern and final free convection stage. Further, free convection in an inclined two-dimensional slit is treated with the fundamental linearization assumptions and also constant temperature gradients in the extemal mass. The physical properties of water are listed for the temperature range from 0 to 100 C.

The above problem of a vertical cylindrical tube was examined experimentally and results compared with theory. Some description is given of the unsteady convection processes and also the transition to turbulence in the experimental column.

The full nonlinear partial differential equations (motion, heat transfer and continuity) are attacked by the method of successive approximations and reduced to an ordinary nonlinear equation. The technique is then applied to the axisymmetric case and the case of diametral antisymmetry. Further, natural convection in a horizontal cylinder and in a spherical cavity are treated and compared qualitatively to temperature profiles obtained by optical techniques. W. H. Schwarz, USA

**5219. Eigenson, L. S., About breakdown in laminar flow of viscous liquid in natural convection** (in Russian), *Teploenergetika* no. 12, 66–69, 1957.

Author derives criteria for transition from laminar flow in the case of free convection and compares them with corresponding values for forced convection. Viscous flow is considered between two parallel vertical plates, between two vertical coaxial cylinders, as well as along a vertical wire and vertical plate.

The method consists in solving simultaneously the approximate flow and heat conduction equations in the laminar boundary layer for specified geometries and evaluating the average velocities and Reynolds numbers. Critical values of the latter for various geometries at which the laminar flow breaks down are obtained by comparing the experimental results expressed in terms of Grashof and Prandtl numbers with theoretical expressions for Reynolds number. Author finds that critical Reynolds numbers are considerably lower for viscous flow corresponding to free convection between parallel vertical plates or coaxial cylinders than for vertical wires or plates. Critical Reynolds numbers differ by small amount for natural convection or forced convection in case of laminar viscous flow along vertical cylinders or plates. B. Zarwyn, USA

**5220. Stewartson, K., On the free convection from a horizontal plate** (in English), *ZAMP* **9a,** 3, 276–282 (Brief Reports), Sept. 1958.

Free convection from a heated vertical plate, and an inclined one, has been treated in the literature. However if the plate is horizontal, the buoyancy has a zero component along the length, and the boundary layer (if one does exist) has a different character than in the inclined cases. Paper examines the boundary layer on a horizontal plate that is heated and faces downward (or cooled and faces upward) and indicates that the solution is in fair agreement with experimental results. If the plate is heated and faces upward (or is cooled and faces downward) then the boundary-layer problem is improperly posed. F. V. Pohle, USA

**5221. Ostroumov, A. G., and Ostroumov, G. A., Problem of thermoelectric convection** (in Russian), *Zh. Tekh. Fiz.* **26,** 3, 636–639, 1956; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 3040.

Authors describe experiments carried out for the purpose of investigating thermoelectric convection phenomena, forming part of the change of the process of the convection-flowing around a body when placed in an electric field. Authors studied the convection heat exchange of a warmed filament with a surrounding medium by means of the optical method. The electric field was created by the supply to the filament and the counter electrodes of stresses, changing in a wide range (from 0 v. to 5830 v.). The experiments showed that the change in the process of free convection-flowing around a body when placed in an electric field is not due to the diminution of the dielectric constant of the air with increase of its temperature, but to the linkage with the presence of a conductivity current, emerging at the expense of a coronary discharge. The phenomenon investigated by the authors is "an electrical wind," although change of the dielectric constant may play a part.

Curves are given for the family of isolines of the discharge current, which were obtained in the experimental work.

V. A. Sukhnev

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*Translation, courtesy Ministry of Supply, England*

**5222. Thomas, P. H., The absorption of radiant heat by fire-fighting foam,** *J. Appl. Chem.* **9,** 5, 265–268, May 1959.

Measurements of the resistance of fire-fighting foams to radiant heat which have previously been reported are discussed in terms of the reflectivity of foam. It is possible to obtain a simple correlation for most of the data which is in reasonable agreement with a calculated correlation. From author's summary

**5223. Townsend, A. A., The effects of radiative transfer on turbulent flow of a stratified fluid,** *J. Fluid Mech.* **4,** 4, 361–375, Aug. 1958.

Criteria are derived theoretically for the subsidence of turbulence due to the action of gravity for thermally stratified flow that is not influenced by the presence of solid boundaries. A new feature in the analysis is that radiative heat transfer is taken into account. Since this tends to reduce temperature inequalities, its effect is to increase the critical value of the Richardson number. With weak radiative transfer turbulence subsides at values of the flux Richardson number that are smaller than one, with strong radiative transfer the critical value of the flux Richardson number equals one.

The flux Richardson number and the Richardson number depend on the logarithmic rate of radiative cooling. A theoretical expression is derived for this quantity, and particular forms are developed for a fluid that is effectively transparent or effectively opaque over distances of the order of the scale of turbulent motion.

Application to atmospheric motion is discussed; it is shown that critical gradients of potential temperature, below which turbulence is possible, can be increased by an order of magnitude due to the presence of water vapor, carbon dioxide and ozone in the upper stratosphere and by a factor two to seven by the presence of water vapor near sea level.

Reviewer thinks that this paper presents an important contribution to the theory of the transition from turbulent to laminar motion in thermally stratified flow.

D. A. DeVries, Netherlands

**5224. Povarin, P. I., and Semenov, S. T., Investigation into critical boiling of water heated below the temperature of saturation when flowing at high velocity in pipes** (in Russian), *Teploenergetika* no. 4, 72–79, Apr. 1959.

Paper describes the results of an investigation of the boiling of water heated below the saturation temperature flowing through the pipes of 3-mm diameter. The relationship $q_{cr}$ is given for the velocity of flow $w$ and the value of underheating ($\Delta t_u = 0 - 200°C$) under pressures in the circuit of 35 atm and $w = 1 - 45$ metres per sec. From authors' summary

**5225. Miropolsky, Z. L., Shitzman, M. E., Mostinsky, I. L., and Stavrovsky, A. A., The effect of inlet conditions upon critical thermal flows of boiling water in pipes** (in Russian), *Teploenergetika* no. 1, 80–83, Jan. 1959.

Paper presents the results of an experimental investigation concerning the effect of throttling upon the value of critical thermal flow at forced motion of a steam-water mixture in pipes with different lengths of heated section. From authors' summary

**5226. Dementiev, V. M., Problems of a boiling layer in aerodynamics** (in Russian), *Teploenergetika* no. 1, 50–56, Jan. 1959.

The theoretical interpretation of boiling is offered for a fine-grained material. On the basis of this theory and the experimental data presented, formulae are derived for determination of the following: the rate of beginning to boil, the resistance of boiling layer and its expansion, the current value of the porosity and the value of clear space, as well as the ratio of the rate of boiling to the rate of the beginning to boil. From author's summary

**5227. Averin, E. K., and Kruzhilin, G. N., Influence of surface tension and viscosity on the conditions of heat exchange when water boils** (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 10, 131–137, 1955; *Ref. Zh. Mekh.* no. 4, 1958, Rev. 4254.

Experiments were carried out at atmospheric pressure and natural convection of water on a copper tube 160 mm long and 8 mm in diameter, and placed horizontally in a large space. Heating was carried by direct current. The surface tension of the water $\sigma$ was varied from $6 \times 10^{-3}$ to $2.9 \times 10^{-3}$ by means of a solution of isoamyl alcohol ($C_5H_{12}O$). The viscosity of the water was changed by dissolving in the water beet sugar from $28.8 \times 10^{-6}$ to $340.7 \times 10^{-6}$ Kg sec/m$^2$. Determinations were made for the critical thermal loading $q_{cr}$ Kcal/m$^2$/hr and the value of the coefficient of heat emission $\alpha$ Kcal/m$^2$/hr °C at a thermal load of 200,000 Kcal/m$^2$/hr. It was established that the coefficient of heat emission $\alpha$ changes proportionally to the surface tension to a degree of $-0.333$ and of the viscosity of the liquid to a degree of $-0.45$. The critical thermal load $q_{cr}$ changes approximately proportionally to the surface tension to a degree of 0.25 and to the viscosity of the liquid to a degree of $-0.2$. A more precise form is proposed for Kruzhilin's criterial relation for heat exchange during boiling, and a formula is given for the calculation of the coefficient of heat emission $\alpha$. It was established that when water is boiled in a large space in atmospheric conditions, when the tubular surface of the heating element made from various technical materials (steel, copper, nickel plated copper) is placed either vertically or horizontally, the critical thermal load lies within the limits of $1.2 \times 10^6$ and $1.3 \times 10^6$ Kcal/m$^2$/hr. P. I. Povarnin

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5228. Brauer, H., Heat transfer laws for film condensation of pure vapours on vertical surfaces** (in German), *Forsch. Geb. Ing.-Wes.* **24,** 4, 105–117, 1958.

Author uses previous results on the flow and heat transfer in three types of falling liquid films [AMR **10** (1957), Rev. 1925] to calculate the thermal resistance of corresponding types of condensate films. The transitions between the three regimes are elucidated. The results agree satisfactorily with known experimental data. The equations for the second regime (laminar film with ripples) appear to be of particular interest. The formulas can be applied to determine the length and the number of tubes wanted to attain a given rate of condensation.

H. A. Vreedenberg, Holland

**Book—5229. Oprecht, U., Regenerators for small gas turbines [Regeneratoren für Kleingasturbinen],** Zürich, Verlag Leemann, 1957, 79 pp. (Paperbound)

The aim of this thesis (ETH, Zürich) is the optimization of the heat exchanger of small open-cycle gas turbines (< 500 hp) by calculation of heat transfer and pressure drop of mainly two types of regenerative heat exchangers with wire screen heating elements and with channel-type elements (flat and corrugated plates)—a recuperative heat exchanger is not considered because of its bulkiness and weight. The effect of carry-over losses and of leakage on turbine performance is taken into account. The wire-screen type is considered more favorable, and author's measurements with various types of screens (varying wire gauge, distance, position of screen in case of two screens, etc.) are reported. Design of sealing devices is briefly discussed and preference is given to the tightly scrubbing type. From a practical point of view the effect of deposits on pressure drop and the difficulties of sealing a heat-distorted rotor are possibly slightly underestimated and re-

quire verification of the theoretical results by assessing actual performance data under operating conditions.

W. Gumz, Germany

**Book—5230. Hryniszak, W., Heat exchangers, applications to gas turbines,** New York, Academic Press, Inc., 1958, xii + 343 pp. $10.

Author stresses the thermodynamic and mechanical design of the heat exchanger in terms of its effect on the suitability, economy and performance of the gas-turbine unit. Specifically, emphasis is placed on the effect of the heat exchanger design on each of the other components; namely, their type, shape, arrangement and performance within the framework of the whole unit. The book comprises fifteen chapters. An introductory chapter discusses the general philosophy of the book. This is followed by three chapters concerning types of and thermodynamic properties of gas-turbine heat exchangers. Chapter five discusses the influence of the heat exchanger on the performance of the gas-turbine unit. The next three chapters are concerned with the design of air preheaters, recuperators and regenerators. Chapters nine and ten consider part load performance of an air preheater and the influence of the air preheater on the governing of the gas-turbine unit. The next four chapters consider, respectively, testing, stress and materials, cost factors, and weight considerations. The last chapter is concerned with general overall design problems. Author has drawn heavily from many excellent texts, his own experience and the extensive practices and experiences of the C. A. Parsons and Co., Ltd. by whom he is employed. The book is generally well written and is recommended for those desirous of becoming better informed about engineering design practice for gas-turbine heat exchangers and, more specifically, of the design practice of one of the best-known British companies.

R. J. Mindak, USA

**Book—5231. Alami, R., and Ageron, P., Extraction and utilization of heat from nuclear reactors, Vol. 3 [Evacuation et recuperation de la chaleur des reacteurs nucleaires, Tome 3],** Paris, Dunod, 1958, ix + 246 pp. 2900 F.

Book is the third of a series intended for engineers working occasionally in the nuclear field. There are four main chapters with following subject matter: First (150 pages), heat-transfer principles; second (40 pages), thermodynamic cycles of converting heat energy into mechanical work; third (30 pages), description of thermal circuits of some existing reactors; and fourth (15 pages), thermal stress.

Treatment is elementary, of handbook style, and often lacks preciseness. Some letter symbols are nowhere defined. At best it is a fair compilation of materials from published sources. Like all French books, there is no index. L. S. Dzung, Switzerland

**5232. de Vries, D. A., Simultaneous transfer of heat and moisture in porous media,** *Trans. Amer. Geophys. Un.* **39,** 5, 909-916, Oct. 1958.

Paper extends a previous approach [Philip & de Vries, *Trans. Amer. Geophys. Un.* **38,** 222-232, 1957] and develops the simultaneous differential equations for the transfer of heat and moisture in porous media under the combined influence of gravity and gradients of temperature and moisture content. In particular a consistent distinction is made between changes of moisture content in the liquid and the vapor phase. The interaction between heat and moisture transfer in steady-state heat conduction is discussed in detail, using numerical values for two soils, a clay loam and a medium sand. The behavior is found to be dependent on the boundary conditions for moisture transfer, on the direction of the temperature gradient, and on the ratio of two moisture diffusivities entering in the analysis. Graphs are given.

From author's summary by G. Sestini, Italy

**5233. Stuhlbarg, D., The design of heating coils for storage tanks,** ASME-AIChE Joint Heat Transfer Conf., Evanston, Ill., Aug. 1958. ASME Pap. 58-HT-1, 24 pp.

A collection of useful information and some practical experience on the subject. A number of separate cases are considered.

D. M. Vestal, Jr., USA

**5234. Grootenhuis, P., The mechanism and application of effusion cooling,** *J. Roy. Aero. Soc.* **63,** 578, 73-89, Feb. 1959.

An attempt has been made to correlate published experimental data on effusion cooling in such a manner that it will be ready for use by the designer. The energy transfer between porous material and coolant was considered in detail. The heat-transfer coefficients were obtained by experimental means. An approximate analysis of the heat insulating effect has been developed, and applied to experiments involving porous plugs installed in the side of a duct carrying hot gases, and porous cylinders swept by hot gases. The analysis applies quite well to results of the author's experiments, and to other published data. The application of effusion cooling in the design of gas turbine blading, combustion chamber linings, rocket motors and the outer skin of rockets is presented. The paper is recommended for those dealing with the design of systems when effusion cooling is required.

G. A. Hawkins, USA

**5235. Capey, E. C., and McKenzie, K. I., Theoretical analysis of the heating of a composite slab, with applications to the kinetic heating of an aircraft wing,** *Aero. Res. Counc. Lond. Curr. Pap.* 412, 28 pp., 1958.

The transient temperature distribution is derived for a composite slab, heated by raising the air temperature on one side at a constant rate to a maximum. Numerical results are evaluated on a digital computer from the theoretical solution, which is obtained in series form. Results are obtained for the temperature difference across the skin of an aircraft wing covered by insulative material; these results show that the effect of the insulative material could be calculated with sufficient accuracy by neglecting its heat capacity. The computer programs are applicable to any problem in which there is no heat transfer at the cold surface.

From authors' summary by S. Lampert, USA

## Combustion

(See also Revs. 5222, 5300)

**Book—5236. Gaydon, A. G., The spectroscopy of flames,** New York, John Wiley & Sons, Inc., ix + 279 pp. $9.

This is the fourth book in a group which was started with a book on "Spectroscopy and combustion theory," first published in 1942 and revised in 1948. Sections of this have recently been covered more fully in "Flames, their structure, radiation and temperature," by Gaydon and Wolfhard, and in "Dissociation energies," by Gaydon. The present volume was written to cover particularly developments based on new techniques for controlling flames and studying their spectra or on new methods for exciting and studying flame-type spectra.

Several chapters are devoted to experimental methods for recording and studying spectra, to infrared techniques, to flame spectrophotometry for determining concentrations of reacting species, and to determinations of flame temperatures from spectra. Other chapters cover special techniques such as "flat" flames and low-pressure flames, flash photolysis, and shock-tube excitation, as well as special situations such as explosions, internal combustion engines, and solid propellant flames.

Results of spectroscopic observations are summarized for a number of flame systems, with particular attention to hydrogen, carbon

monoxide, and hydrocarbon and other "Organic" flames and to flames containing nitrogen, halogens, sulfur, and inorganic substances. A good bibliography is supplied for those who may be interested in additional details.

There are also good discussions of the fundamental theories of spectra and of the interpretation of results in terms of mechanisms of excitation and reaction in flame propagation and in such related processes as carbon formation.

Both the specialist and the more casually interested reader should find this an interesting and stimulating survey of methods of observing and interpreting flame spectra and of the significance of recent results in various aspects of combustion theory and practice. R. C. Anderson, USA

**5237. Essenhigh, R. H., and Woodhead, D. W., Speed of flame in slowly moving clouds of cork dust,** *Combustion and Flame* **2,** 4, 365-382, Dec. 1958.

Flame speeds in cork-dust clouds falling freely in a vertical tube 17 ft long, and either 2 or 3 in. in diameter, were measured as a function of the dust concentration by a photographic method. At the top of the tube, cork dust of two levels of fineness was delivered by a mechanical feeder; except for the dust chute the top was closed, and the bottom could be closed or open as desired. With ignition at the bottom when open the flame travelled up the tube at a constant speed in the approximate range of 40 to 100 cm/sec. The lower limit of inflammability, and the flame speed at that limit, were both found to be independent of dust fineness and of tube diameter within the ranges examined. The flame speed increased with dust concentration up to a maximum whose value increased with fineness and tube diameter. The concentration that gave the maximum flame speed was, for the coarser dust sample, about four times that of the estimated stoichiometric, and for the dust sample with fines, about seven times. The flame length was found to vary linearly with the burning time.

Observations were also made of: (1) a flame propagation downwards, also in "open-to-closed" conditions, that was found to be steady and often to move even more slowly than the falling dust (about 45 cm/sec), and (2) a more rapid explosion flame ("closed-to-open" conditions) in both upward and downward directions. The latter flame accelerated rapidly, usually to above 2 m/sec and tended to be oscillatory. From authors' summary

**5238. Courtney, W. G., Ignition by flow over hot surfaces,** *Jet Propulsion* **28,** 12, 836-837 (Tech. Notes), Dec. 1958.

**5239. Pavlov, S. M., Stabilization of the flame in gas flows of high velocity** (in Russian), *Naucbno-tekbn. Inform. Byul. Leningrad Politekbn. In-ta* no. 5, 22-29, 1957; *Ref. Zb. Mekb.* no. 4, 1958, Rev. 3811.

Paper is a theoretical investigation of the problem of the stabilization of a flame by means of stabilizers of poor flowing-around properties. It is assumed that the burning mixture moves at the speed of the surrounding basic flow of combustible mixture. Again, assuming an ideal blending of the combustible mixture with the products of combustion and that the combustion reaction is describable by the principle of Arkenius and proceeds by the second order, the author obtains a theoretical relation determining the collapse of the combustion. On the basis of this relation and of known experimental data a semi-empirical function is deduced which satisfactorily describes the limits of stabilization of the flame. B. V. Raushenbakh

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5240. Agoston, G. A., Noon, A. W., and Witherly, T. D., Experimental study of a reversed jet flameholder,** *Combustion and Flame* **2,** 4, 333-347, Dec. 1958.

Experiments and analyses were performed to contribute to a basic understanding of the holding of flame in a stream of a combustible gas mixture by a small gas jet directed upstream. The main stream, a mixture of natural gas and air, entered a cylindrical transparent test section at uniform velocity. A flameholding jet of air was directed counter to the stream along the axis of the test section. Observations of the flow field and flame behavior were made by means of time-exposure photographs, high-speed motion pictures, and spark shadowgraphs. A pilot flame was delineated, and its significant time-averaged dimensions were noted as a function of jet and main-stream velocities. New evidence was obtained indicating the occurrence of recirculation. Transmission of flame from the pilot to the main stream is accomplished through a mixture continuously shedding from the pilot flame. Rough burning near a stability limit occurs when significant portions of the shedding mixture are not inflammable. The influence of nonuniform fuel concentration patterns in the main stream on flame stability limits was also investigated. Interpretation of the data leads to conclusions which conflict with a currently held mechanism for reversed-jet flame stabilization. A mechanism is proposed which is plausible in the light of current observations.

From authors' summary

**5241. Sandri, R., On the decomposition flame of liquid ozone-oxygen mixtures in a tube,** *Combustion and Flame* **2,** 4, 348-352, Dec. 1958.

A theory of flame propagation previously developed by the author is applied to a decomposition flame burning above a liquid ozone-oxygen mixture in a tube cooled with liquid oxygen at the outside. Neglecting heat losses to the wall, heat radiation and heat conduction along the glass tube, flame velocities are computed for pressures of 1, 2 and 10 atmospheres for mixtures containing 50, 75 and 100% ozone. The results obtained for atmospheric pressure are in reasonably good agreement with the available experimental result. From author's summary

**5242. Crocco, L., and Grey, J., On the measurement of high frequency stability limits in a liquid bipropellant rocket** (in Italian), *Aerotecnica* **38,** 3, 135-144, June 1958.

**Book—5243. Kuznetsov, L. A., Combustion chambers of fixed gas turbine installations** (in Russian), Moscow-Leningrad, Mashgiz, 1957, 166 pp. + illus. 5r 50k; *Ref. Zb. Mekb.* no. 4, 1958, Rev. 4032.

Book contains fundamental information on the principles of the work of the combustion chambers of fixed gas turbine plant, computation of its effectiveness and of its design outlines. In chapter I author refers to the physico-chemical and operational characteristics of the basic forms of liquid fuels used in Soviet gas turbine plants; he enumerates the principal requirements which have to be met by the system of fuel feed of gas turbine plant working with liquid and gas-forming fuels; information is given on the process of atomizing. In chapter II some concepts are given on the process of combustion, which are essential for the elementary thermo-chemical calculations, as well as for the qualitative judgment of its rational organization. At this point the basic results of the modern theory of flame propagation, and elementary presentations regarding the turbulent diffusion combustion, etc., are introduced. Chapter III deals with the systematic and operational characteristics of combustion chambers. Together with information on the realizable values of the coefficient of air surplus, flow velocity and thermal stresses, etc., facts are supplied regarding the features of the starting system and the different means available for its efficient working.

Information on the accomplished construction of combustion chambers is given in chapter IV. Here, in comparative detail, an account is given of the constructions turned out by the Nevskii

factory and of chambers examined by the Central Boiler-turbine Institute. Data is furnished regarding the velocity and temperature fields, the distribution of atmospheric flows, and so forth. In chapter IV an approximate method is furnished for the calculations for combustion chambers, which is illustrated by an actual numerical example. Determinations are made for air and fuel consumption, the distribution of air flows is ascertained, calculations are carried out for the basic dimensions of a chamber and auxiliary parts, assuring the accomplishment of the required regimes for working.

The book is of undisputed interest and will be useful for engineers and power engineer students. However, it contains a number of inaccuracies which should be corrected. For instance, the calculation for the temperature of the products of combustion (Eq. [53], pp. 96, 151) is carried through without consideration for the thermo-chemical correction factor according to Kirchhof's principle, which fact, at high compression ratios (the target aimed at by modern power engineers), alters the incoming part of the thermal balance by ~ 1 – 1.5%; there are other cases. The presentation of the velocity of flame propagation in a turbulent flow, borrowed by the author from K. U. Shchelkin, should have been compared with the experimental material. G. A. Varshavskii

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5244. Bragg, S. L., and Byworth, S. P. Q., Combustion chamber pressure loss,** *Jet Propulsion* **28,** 12, p. 829 (Tech. Notes), Dec. 1958.

**5245. Uspenskii, V. A., Investigation of the processes of combustion of a gas jet** (in Russian), *Vses. N.-i. In-ta Metallurg. Teplotekhn. Byul. Nauchno-tekhn. Inform.* no. 2, 29–42, 1957; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 2689.

The work was investigated of an industrial burner of the type of a tube within a tube, placed in a chamber of square section. Measurements were made of the field of temperature, the dynamic pressure and the composition of the gas in the longitudinal and transverse sections of the burner. A brief description is given of two possible regimes for combustion, one of which is feasible at small, and the other at high, velocities of the gas (at a constant coefficient for excess air). The quantity variations of the gas motions along and across the burner are investigated.

N. N. Bakhman

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5246. Crocco, L., Grey, J., and Harrje, D. T., On the importance of the sensitive time lag in longitudinal high-frequency rocket combustion instability,** *Jet Propulsion* **28,** 12, 841–843 (Tech. Notes), Dec. 1958.

It was determined experimentally that there is an upper limit to the chamber length at which each mode of longitudinal high-frequency pressure oscillations will occur, and that this limit was accurately predicted for the fundamental mode by Crocco's sensitive time-lag theory over a wide range of mixture ratios. The technique of using longitudinal stability boundaries for simple and direct experimental determination of the sensitive time lag and associated parameters was established as valid and may be used to replace the complicated and difficult chamber transfer function measurements described in previous publications.

From authors' summary

**5247. King, R. O., Hayes, S. V., Allan, A. B., Anderson, R. W. P., and Walker, E. J., The hydrogen engine: Combustion knock and the related flame velocity,** *Trans. Engng. Inst. Canada* **2,** 4, 143–148, Dec. 1958.

A C.F.R. Otto cycle engine modified to prevent the accumulation of lubricating oil in the combustion space was used for the experiments. The relatively weak hydrogen-air mixture required for maximum thermal efficiency was usable at a compression ratio of 20:1 at speeds rising to 1800 rpm. The usable compression ratio diminished as mixture strength was increased and became 14:1 for the nearly correct mixture required for the development of maximum power output. There was none of the backfiring, preignition or detonation that had occurred in earlier experiments in which lubricating oil had accumulated in the combustion space, but combustion knock was not eliminated. Hydrogen-air mixtures are known to burn with exceptionally high velocity and it was shown by indicator diagrams that the observed combustion knock was related to the flame velocity at the temperature and pressure attained by compression in the engine. From authors' summary

**5248. Vandenkerckhove, Jean A., Erosive burning of a colloidal solid propellant,** *Jet Propulsion* **28,** 9, 599–603, Sept. 1958.

With assumptions (1) that at low pressures burning rates are determined by heat transfer within the fizz zone only, (2) that solid propellant decomposition is the rate-controlling step, (3) that erosive burning is due to effects of turbulence in this zone, and (4) that the threshold velocity is that condition when incipient turbulence reaches the zone boundary, the erosion coefficient is calculated.

The conduction equation across the fizz zone is modified by a ratio of the mass burning rate with and without erosion, which is determined from the burning surface temperatures with and without erosion. Fluid dynamic considerations determine the effective height of the fizz zone to be used in the equations.

I. Glassman, USA

## Prime Movers and Propulsion Devices

(See also Revs. 5177, 5229, 5242, 5243, 5247, 5279)

**5249. Portnov, D.A., and Pogodin, S. I., A comparative evaluation of methods of calculation of the characteristics of a transport gas turbine motor** (in Russian), *Trudi N.-i. Labor. Dvigatelei M-va Transp. Mashinostroenie SSSR* no. 3, 5–29, 1957; *Ref. Zh. Mekh.* no. 4, 1958, Rev. 4020.

Calculation formulas are derived and calculations are described for a double-axle transport gas-turbine motor, to cover three cases: utilizing the characteristics of the compressor and the turbine, without the characteristics of the turbine but with those of the compressor, and, finally, without the characteristics of both the compressor and the turbine. In the second and third cases, the parabolic principle is adopted of the change in K.p.d. (efficiency) of the turbine due to the relation of the peripheral velocity on the mean diameter of the turbine wheel to the gas velocity during adiabatic expansion in the turbine. In so doing it is assumed that the gas consumption is not dependent on the number of revolutions of the turbine. Besides, in the third case, the quadratic principle is accepted, postulating the change of the adiabatic work of the compressor in relation to the number of its revolutions. A deduction is made, based on the calculations carried out, that when the number of revolutions of the compressor equals 0.8–1.0 of the calculated, and the number of revolutions of the power-turbine lies in the range of 0.6–0.8 of the calculated, all three methods give approximate (closely connected) results. The change of power of the motor in the above circumstances lies within the limits of 0.4–1.0 of the calculated value.

A. I. Loshkarev

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5250. German, A. P., The thermal coefficient of useful performance of internal combustion motors** (in Russian), *Zap. Leningrad Gorn. In-ta* **34**, 1, 9–36, 1957; *Ref. Zh. Mekh.* no. 4, 1958, Rev. 4028.

A method is put forward for the investigation of the theoretical cycles of internal combustion engines examined as unfastened physico-chemical processes of conversion of the free internal energy of the combustion mixture into useful work, having in view the variability of the physical constants of the mixture when subjected to compression, combustion and expansion. A number of numerical examples are given for the determination of different cycles. It should be noted that a similar method, for some little time back, has been applied to the calculations of the cycles of gas turbine installations.

A. I. Loshkarev
*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5251. Huttner, E., Small centrifugal gas turbines** (in German), *Öst. Ingenieur Z.* **1**, 12, 513–520, Dec. 1958.

**5252. Benson, R. S., Experiments on two-stroke engine exhaust ports under steady and unsteady flow conditions,** Instn. Mech. Engrs., Prepr., 1958, 27 pp.

The objective of the work was to establish suitable data for the design of exhaust ports. Two theoretical methods were used. In the first method the discharge was analyzed by the method of characteristics and it was shown that the discharge was by wave action; the second method neglected the particle velocity in the cylinder.

The experimental investigation was carried out on a special machine which simulated in all respects the exhaust-port arrangement of a two-stroke cycle engine. The following conclusions were drawn: (1) the coefficient of discharge for the exhaust ports increased with cylinder pressure and decreased with increased port area; (2) the coefficient of discharge was lower under dynamic conditions than under steady flow conditions; (3) the dynamic coefficient of discharge decreased with reduction in cylinder length at release; (4) the effect of early port opening was to reduce the blowdown time; and (5) the effect of piston movement during discharge was to reduce the effective port area required.

From author's summary by A. N. Petroff, USA

**5253. Dillaway, R. B., Propulsion systems for space flight,** *Aero. Engng. Rev.* **17**, 4, 42–49, 52, Apr. 1958.

**5254. Schwarzler, K., Hot water rockets as launchers** (in German), *Z. Fluguiss.* **6**, 1, 1–9, Jan. 1958.

## Magneto-fluid-dynamics

(See also Rev. 5161)

**5255. Sears, W. R., and Resler, E. L., Theory of thin airfoils in fluids of high electrical conductivity,** *J. Fluid Mech.* **5**, 257–273, Feb. 1959.

Steady plane flows of incompressible fluid past thin cylindrical bodies are treated with the undisturbed uniform magnetic field taken to be (1) parallel to, and (2) perpendicular to, the undisturbed uniform stream. Only flows with large (or infinite) conductivity are considered. The theory would apply to flows at high "magnetic Reynolds numbers," e.g., to flows involving high gas temperatures or flows of liquid metals.

The linearized equations are derived for flows with infinite conductivity. In case (1), the flow is shown to be irrotational (unless the stream velocity is equal to the Alfven velocity) and current-free except for surface currents at the bodies. In case (2), the flow fluid is composed of a system of waves involving currents and vorticity.

In case (1), flows with large finite conductivity are treated by a boundary-layer approximation, the surface currents being replaced by thin boundary layers of large current density. In case (2), flows with large finite conductivity cause damping of the wave system.

Results are given for forces on sinusoidal walls and on airfoils.

A. W. Babister, Scotland

**5256. Krook, M., Structure of shock fronts in ionized gases,** *Ann. Phys.* **6**, 2, 188–207, Feb. 1959.

Author shows how equations for the electric field and moments of the ion and electron velocity distribution functions can be reduced to a determinate set of ordinary differential equations by assuming that the distribution functions are the sum of two modified Maxwell distributions. A formal method of solution is described, and results are obtained for moments of first and second order. A discussion of these and of solutions for moments of higher order is left to a subsequent paper.

P. G. Saffman, England

**5257. Kruskal, M. D., and Kulsrud, R. M., Equilibrium of a magnetically confined plasma in a toroid,** *Physics of Fluids* **1**, 4, 265–274, July-Aug. 1958.

General properties of plasma in static equilibrium are derived from equations of magnetostatics. These properties are expressed as integrals over surfaces of constant pressures, which are shown to be topologically toroidal under general assumptions. A variational principle for such equilibrium conditions is derived. One of its consequences is characterization of equilibria by value of certain invariants. Conditions are then obtained for steady state of a plasma slowly diffusing across a magnetic field out of a topologically toroidal region.

From authors' summary by O. K. Mawardi, USA

**5258. Kruskal, M. D., and Oberman, C. R., On the stability of plasma in static equilibrium,** *Physics of Fluids* **1**, 4, 275–280, July-Aug. 1958.

Criteria for stability of a system of charged particles are derived from Boltzmann equation in small $m/e$ limit. These criteria are obtained from examination of variation of energy due to a perturbation subject to constraint that all time-independent constants of motion have their equilibrium values. Authors find that first-order variation of energy vanishes, while second-order variation yields quadratic form in displacement variable. Positive definiteness of form is sufficient condition for stability. Authors also state theorem comparing their stability criterion with that of conventional hydromagnetic theory where heat flow along magnetic lines has been neglected.

From authors' summary by O. K. Mawardi, USA

**5259. Johnson, J. L., Oberman, C. R., Kulsrud, R. M., and Frieman, E. A., Some stable hydromagnetic equilibria,** *Physics of Fluids* **1**, 4, 281–296, July-Aug. 1958.

Authors obtain hydromagnetic equilibria for a variety of situations which have approximately zero pressure uniform axial magnetic field. Criteria for ascertaining the stability of these equilibria are derived by means of energy principle. In particular, if helically invariant fields are present, stable equilibria with non-zero pressure and net axial current can be found.

From authors' summary by O. K. Mawardi, USA

**5260. Meyer, F., The stability of a plasma in crossed magnetic fields** (in German) Z. *Naturforsch.* **13a**, 12, 1016–1020, Dec. 1958.

The stability of a plasma in crossed magnetic fields is investigated for the following equilibrium configuration. A plasma with an interior horizontal magnetic field is supported against gravity

by a horizontal vacuum magnetic field which is inclined at some angle $\alpha$ to the interior field. This example is an extension of the case $\alpha = 0$ investigated by Kruskal and Schwarzschild. It is found that for all disturbances with small wavelengths, including those disturbances which give the Kruskal-Schwarzschild instability, stability can be restored by using a non-zero $\alpha$. Perturbations of sufficiently large wavelengths are found unstable for every $\alpha$ and every ratio of field strength. From author's summary

**5261. Kudriavtsev, V. S., Energy diffusion of fast ions in an equilibrium plasma** (in Russian), *Zh. Eksp. Teor. Fiz.* **34**, 6, 1558–1565, 1958. (Translated by Morris D. Friedman, P. O. Box 35, W. Newton, Mass., K-183, 9 pp.).

Author considers the problem of the energy diffusion of fast ions injected into an equilibrium plasma. The initial conditions for the energy distribution of the injected ions is a monochromatic distribution with the values exceeding the mean thermal energy in the plasma. The distribution over the velocity directions is isotropic. The kinetic equation for a completely ionized plasma in the absence of external field is taken from the well-known Landau paper [*Zh. Eksp. Teor. Fiz.* **7**, p. 203, 1937]. The influence of the mutual particle collisions is assumed to be small and the time-variation of the distribution function of these particles is determined by the collisions with the ions and electrons of the equilibrium plasma distributed according to a Maxwellian law. This leads to an expression for the particle flux in the velocity space and to an equation for the energy density distribution. The latter equation is solved by means of the Laplace transform and an approximate assumption for the form of the energy density. The final expression for the energy density describes the energy distribution of the ions injected into the plasma being in an equilibrium state. The result for an arbitrary initial distribution can be found by using the superposition principle since the equations are linear.

M. Z. v. Krzywoblocki, USA

**5262. Nakagawa, Y., Experiments on the instability of a layer of mercury heated from below and subject to the simultaneous action of a magnetic field and rotation, Part II,** *Proc. Roy. Soc. Lond.* (*A*) **249**, 1256, 138–145, Jan. 1959.

Continuation and further elaboration of the experiment reported earlier [AMR **11** (1958), Rev. 2172]. Surface motion of the rotating pool of mercury placed in the magnetic field of a cyclotron magnet is observed and the cellular structure is analyzed. When increasing the magnetic field, a sudden change in scale of the cells is observed at a characteristic value of the nondimensional parameter.

L. S. G. Kovasznay, USA

## Aeroelasticity

(See also Revs. 5098, 5221)

**5263. Gaukroger, D. R., A theoretical treatment of the flutter of a wing with a localized mass,** *J. Roy. Aero. Soc.* **63**, 578, 95–103, Feb. 1959.

Flutter calculations were made on a fixed root model wing carrying a localized mass. Modes were calculated for the wing for certain conditions of restraint at the localized mass position, the choice of modes being based on observation of the behavior of wind-tunnel models under fluttering conditions. The general agreement between the calculations and experimental results was good and justified the extension of the calculations to conditions of localized mass loading that were not covered by experiment. The extended calculations show that there is an optimum spanwise position for a localized mass from the flutter point of view, and that this optimum position is related to the nodal line position in the overtone torsion mode of the bare wing. For values of localized mass parameters likely to occur in practice, flutter speeds can be estimated from calculations in a very limited number of modes, the choice of modes depending on the localized mass loading. From author's summary

**5264. Hedgepeth, J. M., and Waner, P. G., Jr., Application of the method of Stein and Sanders to the calculation of vibration characteristics of a 45° delta-wing specimen,** *NASA Memo* 2-1-59L, 26 pp., Feb. 1959.

Authors have applied the Stein-Sanders method [*NACA TN* 3640] of obtaining influence coefficients for thin low-aspect-ratio wings to a 45° delta wing. Those coefficients are used to obtain the natural modes and frequencies in symmetric and antisymmetric vibration. The resulting frequencies are compared with experimental values and found to be all too high, the error being greater in the higher modes of vibration. It is concluded that the method is not satisfactory for the particular wing structure chosen.

L. C. Woods, USA

**5265. Murrow, H. N., An analysis of flight-test measurements of the wing structural deformations in rough air of a large flexible swept-wing airplane,** *NASA Memo* 12-3-58L, 46 pp., Jan. 1959.

This investigation is part of a series of NASA flight tests to determine the aeroelastic behavior of large airplanes and the importance of different factors involved in gust response problems [cf. AMR **12** (1959), Rev. 334, where the basic methods of measurement and analysis were described]. Wing deflections were measured by means of an optigraph system at various locations on the wing and correlated with vertical gust velocity and local angle of attack. The resultant power spectra and frequency response functions are then compared with those expected from quasistatic loadings, and the amplifications due to the flexibility of the structure can be determined.

Results from a 90-second run at 3000 feet above ground indicate that flexibility reduces the amplitude of deflection and the local angle of attack at low frequencies and amplifies them above about 1 cps with a peak at the wing first bending mode (1.5 cps). The (chordwise-) twist spectra are amplified all over the frequency range. The total (rms) factor of amplification is about 1.05 to 1.10 for wing deflection and about 1.5 to 2.8 for twist.

The frequency response functions presented (range of validity 0.3 to 1.8 cps) provide useful information for comparison with analytical calculations. H. Merbt, Sweden

**5266. Fiorini, V., Aeroelastic models for low-speed test** (in Italian), *Aerotecnica* **38**, 5, 260–268, Oct. 1958.

Flutter investigations for high-aspect-ratio wing and tail plane to a flight speed of 0.8–0.85 Mach number are very convenient with aeroelastic models in a low-speed wind tunnel.

Results of dimensional analysis of the flutter equations are reported, and similarity laws for aerodynamic, elastic and inertial forces are applied to the design and construction of the subject models.

Particularly, a monospar model, in which only the spar has strength attributions and the balsa sections have aerodynamic and inertial functions, is described in detail.

Preliminary test to the flutter investigation in a wind tunnel, such as determination of torsional and flexural rigidity for the spar, positioning of c g and determination of the moment of polar inertia at the c g axis parallel to the elastic axis of the balsa sections, is also summarily described.

From author's summary

**5267. Woodcock, D. L., Calculated aerodynamic forces on a sweptback untapered wing oscillating in incompressible flow,** *Aero. Res. Counc. Lond. Curr. Pap.* 411, 21 pp., 1958.

Airforces have been calculated for an aspect-ratio-2 wing with 40° sweep oscillating in symmetric modes at reduced frequencies of 0, 0.6, and 1.8. Results are presented tabularly as influence

matrix based on displacement at the collocation points, and also as matrices of the overall derivatives for modes of the form $|\eta|^n$; $\eta$ is a dimensionless spanwise coordinate, and $n$ has been taken as 0, 1, 4. Details of the theory are not given, but the method used was the vortex-lattice method modified by using a smaller chordwise mesh size near the leading edge than over the main portion of the wing. The modified lattice results compare favorably with so far unpublished standard lattice calculations based on the Multhopp method. The results of this report will be compared with experiments in progress at the Royal Aircraft Establishment.

H. P. Liepman, USA

**5268. Movchan, A. A., The linear oscillations of a plate moving in a gaseous medium at a high velocity** (in Russian), *Trudi 3-go Vses. Matem. S'ezda*, Vol. 1, Moscow, Akad. Nauk SSSR 1956, p. 206; *Ref. Zh. Mekh.* no. 11, 1957, Rev. 12549.

The problem is examined of the linear oscillations of a plate moving in a gaseous medium. The forces of excess pressure are taken into account by means of an approximate formula. The problem leads to the investigation of the spectrum of natural values of the unself-stressed boundary problem for the equation of the fourth order. The results obtained in regard to the change in natural values with change of the velocity of motion of the plate enable certain deductions to be made on the stability of the oscillations.

From author's summary

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5269. Kemp, R. H., Hirschberg, M. H., and Morgan, W. C., Theoretical and experimental analysis of the reduction of rotor blade vibration in turbomachinery through the use of modified stator vane spacing,** *NACA TN* 4373, 43 pp., Sept. 1958.

Note uses Fourier series to represent aerodynamic loading induced on rotor blades by stator vanes spaced nonuniformly. Fourier coefficients are evaluated numerically in terms of stator blade spacing and displacements. One set of stator group displacements indicated theoretical reductions of 37% in harmonic coefficients over uniformly spaced vanes. Strain gage measurements gave a correlation in amplitude reductions of 37% and 41% in the principal modes. A theoretical reduction in excitation of 68% is predicted for stator displacements of 8.5% of uniform spacing in a 3-segment configuration.

R. L. Leutzinger, USA

## Aeronautics

(See Revs. 5124, 5271)

## Astronautics

(See also Revs. 4868, 4869, 4878, 5246, 5324, 5325, 5327, 5328)

**Book—5270. Ten steps into space,** Philadelphia, Pa., The Franklin Institute (Monograph no. 6), 1958, x + 202 pp. $4. (Paperbound).

Booklet contains ten lectures given in the Spring of 1958 on certain aspects of space flight. Most are of a semipopular, expository nature. They include a short history by Willy Ley, the rocket by K. Stehling, rocket fuels by H. W. Ritchey, satellite instrumentation by S. F. Singer, celestial mechanics by P. Herget, the Explorer and its temperature control in particular by G. Heller, a discussion of the atmospheres of Mars and Venus by J. I. F. King, space medicine by D. G. Simons, and a look at future space travel by I. M. Levitt. Most of these can be easily read by anyone with a bare minimum of technical knowledge; some serve as a general introduction, while others are limited to a narrower aspect. The quality of exposition is good and sometimes excellent; Ley, Herget, Simons and Levitt would certainly come within the latter category.

The tenth lecture by K. A. Ehricke is a technical, mathematical paper entitled "Error analysis of single and two-force field spacecraft orbits." The three types of error considered are in the velocity, position and timing, and the effects of these upon take-off orbit from the earth and upon transfer orbits are analyzed in detail. The not surprising conclusion is that for interplanetary flights some means of corrective maneuvering will be necessary.

R. E. Street, USA

**Book—5271. Spitz, A., and Gaynor, F., Dictionary of astronomy and astronautics,** New York, Philosophical Library, 1959, vi + 439 pp. $6.

Concise definitions of over 2,200 terms and concepts related to astronomy and astronautics—a wide gamut of terms ranging from those as old as man to those as new as the Space Age on the threshold of which he now stands.

From preface

**5272. Leikin, G. A., On the influence of tidal friction on the motion of the earth's satellite** (in Russian), *Astron. Zh.* **35**, 2, 297–300, 1958. (Translation by Morris D. Friedman, Inc., P. O. Box 35, W. Newton, Mass., L-121, 6 pp.)

Author presents a simplified theory and technique for estimating tidal friction acting upon a satellite of the earth orbiting close to the surface. The friction is assumed to arise from the earth completely covered by an ocean of uniform depth. The theory follows but slightly modifies the similar problem considered by Lamb. The moment induced by the tidal wave is related then to the rate of change of translational energy of the satellite, and, with certain approximations, this energy loss is related to the time of fall from a particular altitude to the surface of the earth. The effects of atmospheric drag are neglected.

Two sample computations are included. As might be expected, the mass of the satellite must be very great for tidal effects to become important. For example, a satellite of mass 1000 kg requires more than $4 \times 10^{18}$ years to fall to earth from an altitude of 1000 km.

W. W. Berning, USA

**5273. Small, B. B., The earth satellite orbit,** *Astronautics* **4**, 1, 40–41, 47–48, Jan. 1959.

**5274. Baker, R. M. L., Jr., Note on interplanetary navigation,** *Jet Propulsion* **28**, 12, 834–835 (Tech. Notes), Dec. 1958.

**5275. Rinehart, J. S., Meteorites, satellites, and ceramics,** *Bull. Amer. Ceram. Soc.* **37**, 11, 461–467, Nov. 1958.

**5276. Miele, A., and Cavoti, C. R., Generalized variational approach to the optimum thrust programming for the vertical flight of a rocket, Part II: Application of Green's theorem to the development of sufficiency proofs for particular classes of solutions** (in English), *Z. Flugwiss.* **6**, 4, 102–109, Apr. 1958.

For a rocket in vertical flight the variational problem of the optimum burning program leads to extremal solutions composed of zero-thrust subarcs, subarcs flown with maximum engine output and variable-thrust subarcs. The analytical problem is such that the Legendre-Clebsch condition fails to provide any information on the minimal or maximal character of all subarcs. An analogous difficulty is encountered when applying the more stringent Weierstrass condition at points of the variable-thrust subarcs.

From authors' summary

**5277. Skiyanskii, A. L., Special trajectories during impact by pairs in the generalized problem of three bodies** (in Russian), *Ukr.*

*Matem. Zb.* **9**, 2, 163–175, 1957; *Ref. Zh. Mekh.* no. 4, 1958, Rev. 3670.

The problem is investigated of three material points interacting with forces $m_i m_j f(r_{ij})$, where $m_i$ and $m_j$ are the point masses, $r_{ij}$ is the reciprocal distance and $f(r)$ is the holomorphous function for all positive values for $r$, satisfying the condition $\lim r^{-2n+1} f(r) = l \neq 0$ when $r \to +0$. The paper appears to be a generalization of Yu. D. Sokolov's investigations ["Special trajectories of a system of free material points," Part I, Akad. Nauk, USSR, 1951] and concerns the properties of a motion near an impact in pairs of two out of three bodies. G. N. Duboshin

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5278. Lisovskaya, M. S., The trajectories of the flights of rockets around the moon** (in Russian), *Bynl. In-ta Teor. Astron., Akad. Nauk SSSR* **6**, 8, 550–565, 1957; *Ref. Zh. Mekh.* no. 11, 1957, Rev. 12382.

The problem is set of constructing trajectories for a rocket released from the Earth, circumnavigating the Moon at a small distance from it and returning to Earth without waste of fuel en route. The possibility is indicated of constructing suitable trajectories with arbitrarily small minimum distances from the Moon on the condition that the rocket velocity relative to the Earth at the moment of approach to the Moon is sufficiently large and opposite to the direction of the Moon's velocity. Using a method of graphical integration devised by the author, five orbits are constructed having an axis of symmetry. These orbits are divided into two types. Orbits of the first type form two loops, one of which encompasses the Earth, the other the Moon. Orbits of the second type form one loop encompassing the Earth as well as the Moon.

G. A. Merman

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5279. Swanson, A. G., A five-stage solid-fuel sounding-rocket system,** *NASA Memo* 3-6-59L, 22 pp., Mar. 1959.

A description is presented of a five-stage solid-fuel sounding-rocket system which can boost payloads of 25 and 100 pounds to altitudes of 525 and 300 nautical miles, respectively. Data obtained from a typical flight test of the system are discussed.

From author's summary

**5280. Friedman, H., Rocket astronomy—window into space,** *Astronautics* **4**, 1, 26–27, 69–70, Jan. 1959.

**5281. Instant rocketry,** *Astronautics* **4**, 1, 28–29, 82–85, Jan. 1959.

**The following papers are from the 16 papers presented at a Discussion on observations of the Russian artificial earth satellites and their analysis, on Nov. 29, 1957 under the leadership of H. S. W. Massey and published in** *Proc. Roy. Soc. Lond.* (*A*) **248**, 1252, 1–87, Oct. 1958.

**5282. Jacchia, L., Method for computations of satellite orbits,** 43–44.

**5283. Burt, E. G. C., The computation of orbit parameters from interferometer and Doppler data,** 48–55.

**5284. King-Hele, D. G., Perturbations of the orbit of a satellite near to the earth,** 55–62.

**5285. Evans, J., Vibration and acoustical problems in missiles,** *Noise Control* **4**, 6, 22–26, Nov. 1958.

# Acoustics

(See also Revs. 4968, 5013, 5210, 5285)

**5286. Jacobson, M. J. Correlation of a finite distance point source,** *J. Acoust. Soc. Amer.* **31**, 4, 448–453, Apr. 1959.

The direction of a sound or a noise source relative to two receivers can be measured by finding the delay in one of the receivers which makes the correlation of the two signals a maximum. From this delay and the speed of sound the direction can be calculated by simple geometry, assuming the distance to the noise source large as compared to the distance between the two receivers.

In this article the more general case is treated where the distance $r$ of the noise source to the receivers is not large as compared to the distance $d$ between the two receivers. The error which is committed by assuming $r/d$ large is plotted for various angles $\theta$ of the noise direction. For $r/d > 1$ the error $E$ is a maximum when $\theta$ is 45° and can be expressed in degrees approximately by $E \sim 3.58\, d/r°$. H. P. Eichenberger, USA

**5287. Kawai, N., On the acoustic field of a vibrating, long thin ribbon. Part I,** *J. Phys. Soc. Japan* **13**, 11, 1374–1384, Nov. 1958.

Rigorous two-dimensional solutions are given for the acoustic field of an infinitely long thin ribbon, whose vibrations are normal to its plane, and depend only on the transverse coordinate. Rectangular coordinates and an expansion in hypergeometric polynomials are used. The cylindrical solution at large distance, pressure on the ribbon, and radiated power, are obtained as a function of wave number. Note omission in Eq. (18).

From author's summary by P. Rudnick, USA

**5288. Beranek, L. L., The transmission and radiation of acoustic waves by structures** (The 45th Thomas Hawksley Lecture), Instn. Mech. Engrs., Prepr., 26 pp., 1958.

In this lecture, author treats the problem of acoustic transmission through walls and panels in the audible frequency range. At low frequencies the panel vibrates as a plate or a stretched membrane. At higher frequencies the panel may behave as a quasi-infinite sheet. Above a particular "critical" frequency, the wavelength of the bending waves in the panel will be longer than the compressional waves in air at the same frequency. The two wavelengths may be brought into coincidence provided the airborne wave impinges on the panel at an angle $\theta$ determined by

$$\cos \theta = c_a / c_b$$

where $c_a$ and $c_b$ are the speeds of sound in the air and panel, respectively. At the coincidence angle, an airborne wave striking one side of a panel will set it into a level of vibration such that the magnitude of the airborne wave radiated from the other side may be only a few decibels below that of the incident wave. This effect is discussed both for airborne waves impinging at individual angles of incidence and for waves at many angles of incidence simultaneously.

Measurements on plates, concrete sheets, and masonry walls is presented and analyzed. Analysis of the different types of response to airborne wave excitation is made. Rules for the selection of simple and complex structures for buildings and vehicles are suggested.

Means for reducing the response of structures to airborne wave and mechanical excitation include the use of damping materials in or on the structure, the introduction of discontinuities and the use of sound-absorbing blankets in the structure. Recent data on damping materials and means for utilizing them to produce maximum reduction of flexural waves is presented. It is shown that by proper utilization of such materials a structure-borne wave may be

attenuated in a given distance by a factor of 10 or more than when utilized in a conventional manner.

From author's summary

**5289. Embleton, T. F. W., and Thiessen, G. J., Efficiency of a linear array of point sources with periodic phase variation,** *J. Acoust. Soc. Amer.* **30,** 12, 1124–1127, Dec. 1958.

The radiation efficiency of a uniform linear array of point sources with periodic phase variation is evaluated. Two different types of interference are found, depending on whether the characteristic length of the phase variation is greater or less than the wavelength of the sound radiated. In its application to suction roll silencing in paper mills it is found to be less effective than a linear continuous phase variation.

From authors' summary

**5290. Riollet, G., Noise generation by a fan treated from the point of view of dimensional analysis** (in French), 9th Congrès Intern. Mécan. Appl., Univ. Bruxelles, 1957; **2,** 448–458.

With the aid of dimensional analysis author looks for laws concerning the sound radiated by geometrically similar fans working with similar fluid-velocity fields. He finds that the sound intensity (Watt/m$^2$) radiated by the fan only is a function of Mach number, discharge coëfficient $Q/UD^2$, and Reynolds number. The influence of *Re* number, in author's opinion, is negligible. Formulas usually found in textbooks on sound radiated by fans and based on experiments do not contradict the results of author.

J. H. van de Stadt, Holland

**5291. Elder, S. A., Cavitation microstreaming,** *J. Acoust. Soc. Amer.* **31,** 1, 54–64, Jan. 1959.

An attempt has been made to discover by experiment what physical assumptions and approximations are appropriate in the theory of cavitation microstreaming, especially for cavitation bubbles located near solid boundaries. A systematic investigation of the phenomenon has been made and its dependence on certain parameters (e.g., amplitude of sound) has been determined. The investigation has disclosed that as the sound amplitude is varied, other conditions remaining the same, the streaming changes discontinuously through several stable regimes. It appears that in order to account for the generation of vorticity one needs to assume different conditions at the boundaries for each regime. For at least one regime, a theoretical model due to Nyborg seems to be applicable; comparison is made with experimentally determined streaming velocities.

From author's summary by W. P. Mason, USA

**5292. Wada, Y., Attenuation of ultrasonic waves in suspensions of viscoelastic spheres,** *J. Phys. Soc. Japan* **13,** 11, 1390–1398, Nov. 1958.

The attenuation of ultrasonic waves for suspensions of polymethyl methacrylate particles (mean radius $a$ = ca. $40\mu$) in ethyl alcohol, carbon tetrachloride and their mixtures is measured in the range of frequency 1 – 6 Mc/s. For $ka > 0.5$ ($k = 2\pi/$wavelength) the scattering and reflection losses are found to be predominant origins of the attenuation. But when $ka$ approaches unity, the observed attenuation lies markedly below the prediction from Rayleigh's formula which is established under the condition of $ka \leqslant 1$. For $ka < 0.5$, the viscous loss predicted by Lamb and Urick and the loss due to the internal viscosity of suspended particles become appreciable. A simple theory for the latter loss is developed under some assumptions. The sum of the theoretical values of these three kinds of losses is in satisfactory agreement with the experimental one. In the frequency range where the internal loss predominates, the method employed in this research can be used to measure the volume viscosity of a solid material.

From author's summary

**5293. Dunn, F., and Fry, W. J., Precision calibration of ultrasonic fields by thermoelectric probes,** *Inst. Radio Engrs. Trans.* **PGUE-5,** 59–65, Aug. 1957.

The highly stable, small and readily constructed ultrasonic probe, developed and in use for the past five years, is discussed from the point of view of construction, calibration and operation. This transient type, thermoelectric probe yields information concerning the pressure amplitude, particle velocity amplitude and intensity of the ultrasonic field in which it is placed. If the field characteristics are known, the principle of operation of the probe provides a method for determining the absorption coefficient of minute quantities of material.

From authors' summary

**5294. Kamps, E. C., Statistical evaluation of near-field sound pressures generated by the exhaust of a high-performance jet engine,** *J. Acoust. Soc. Amer.* **31,** 1, 65–67, Jan. 1959.

The operation of modern, high-performance turbojet engines creates an environment that promotes rapid structural fatigue in many areas in jet aircraft. To facilitate the simulation of a valid test environment and, therefore, enable fatigue-resistant structure to be readily developed, the characteristics of the near-field sound pressures created by the jet stream must be established. This study is concerned with the establishment of the level and the rate of occurrence of peak pressures in relation to the commonly measured rms level. A method is presented which enables the determination of these qualities from rms sound pressures recorded on magnetic tape. Also included are typical results obtained from near-field sound pressures generated by the jet exhaust of a General Electric CJ805 turbojet engine. Noted conclusions: (1) Occurrence of peak pressures does not follow a Rayleigh distribution. (2) Peak pressure distribution is not altered by the physical position related to the jet stream nor does it appear to be a function of frequency. (3) A maximum ratio of peak to rms pressure is shown to exceed four, but a physical limit is not established. From author's summary

**5295. Greenspan, M., and Tschiegg, C. E., Tables of the speed of sound in water,** *J. Acoust. Soc. Amer.* **31,** 1, 75–76, Jan. 1959.

An equation and tables for the speed of sound in water are reprinted from a recent paper published in *Journal of Research, National Bureau of Standards.* From authors' summary

**5296. Nomoto, O., Deviation from linearity of the concentration dependence of the molecular sound velocity in liquid mixtures,** *J. Phys. Soc. Japan* **13,** 12, 1524–1528, Dec. 1958.

Accuracy of the linear dependence of the molecular sound velocity on the molar fractions of the component liquids was investigated in about 100 binary mixtures, employing existing ultrasonic data. The linearity proved to hold true within an accuracy of 0.3% for mixtures of two hydrocarbons or two monovalent alcohols, among others, while the deviation from linearity was larger (0.3%–4%) for the following mixtures: the aqueous solutions, mixtures containing one heavy component such as carbon tetrachloride or chloroform with light partners, mixtures containing glycerine, glycol, aniline, acetone, nitrobenzene, etc. The molecular compressibility (of Wada) showed a similar tendency, though the deviation from linearity was somewhat smaller than that of the molecular sound velocity, especially in case of the mixtures consisting of two component liquids of widely different densities. From author's summary

**5297. Nomoto, O., Empirical formula for sound velocity in liquid mixtures,** *J. Phys. Soc. Japan* **13,** 12, 1528–1532, Dec. 1958.

An empirical formula $V = (R/v)^3 = \{(n_A R_A + n_B R_B)/(n_A v_A + n_B v_B)\}^3$ was derived for the sound velocity in binary liquid mixtures consisting of two component liquids $A$ and $B$, on the assumption of the linear dependence of the molecular sound velocity

$R = vV^{1/3}$ ($v$ = molar volume and $V$ = sound velocity) on concentration (molar fractions $n_A$, $n_B$) and the additivity of the molar volume. The suffixed quantities correspond to the component liquids. This formula holds true for such mixtures for which the linearity of the molecular sound velocity and the additivity of the molar volume is comparatively good, the mixtures of two hydrocarbons and those of two monovalent alcohols being the examples. The values of the empirical sound velocity were compared with those calculated by this formula for 38 mixtures with comparatively small values of the percentage deviation from linearity of $R$, and 28 of them showed deviations not larger than 1% in $V$. Of the 31 liquid mixtures with $\Delta R/R \leqq 0.3\%$, 5 showed deviations of $V$ larger than 1%, 4 of them being the mixtures consisting of two component liquids of widely different molar volumes.

From author's summary

**5298. Kosten, C. W., and Janssen, J. H., Acoustic properties of flexible and porous materials** (in English), *Acustica* **7**, 6, 372–378, 1957.

The theory of Zwikker and Kosten for the propagation of sound waves in porous, flexible materials is extended by the introduction of (known) expressions for the complex density and bulk modulus of air in cylindrical pores. The solution of the equations for the case of a flexible layer on a rigid wall is simplified, discussed, and compared with measurements. Reviewer's note: reference may also be made to the work of Biot [*J. Acoust. Soc. Amer.* **28**, 168–191, 1956; AMR **9** (1956), Revs. 3190, 3191].

H. Deresiewicz, USA

## Micromeritics

(See also Revs. 4978, 5078, 5085, 5116, 5173)

**5299. Walkenhorst, W., A simple method for generating aerosols having particle sizes below 0.1 micron** (in German), *Staub* **19**, 1, 12–13, Jan. 1959.

A process for generating an aerosol by condensation of tungsten oxide is described. If the condensation is carried out under certain controlled conditions, an aerosol will be produced having particle sizes below 0.1-micron size. The particle-size distribution is determined by photographs taken with the electron microscope, which show the most frequent particle size of 0.015 micron, and about 10% of the particles up to 0.008 micron. The apparatus for the evaporation of tungsten oxide is illustrated and described; a sample taken with thermal precipitator photographed with electron microscope is shown; and the corresponding particle-size histogram is given. The number of particles per c.c. is about $13 \times 10^4$ in the sample given, which can be diluted to the desired concentration.

K. J. DeJuhasz, Germany

**5300. Williams, F. A., Spray combustion and atomization,** *Physics of Fluids* **1**, 6, 541–545, Nov./Dec. 1958.

For the description of the complex disorder encountered in sprays a statistical approach is required. A description of the behavior of sprays is presented, which includes the effect of droplet growth, the formation of new droplets, collisions, and aerodynamic forces. Criteria for the efficiency of impinging jet atomization are developed. It is shown that, if the incident jets have a size distribution of a generalized Rosin-Rammler type, the resulting spray belongs to the same class of distributions. The size history of evaporating sprays is also obtained from the theory. A spray combustion analysis given by Probert is extended to include more general size distributions, the effect of droplet interactions, and the relative motion of the droplets and the fluid. It is shown that the overall spray evaporation rate is largest for uniform sprays.

This paper is an attempt at a unified spray theory, in a very condensed form, the value of which for the reader would be enhanced by more detailed explanations of the concepts used, and of the various mathematical operations.

K. J. DeJuhasz, Germany

**5301. Gallily, I., and La Mer, V. K., On the behavior of liquid droplets after impinging on solid surfaces,** *J. Phys. Chem.* **62**, 10, 1295–1299, Oct. 1958.

Deposition of particles impinging on solid surfaces was investigated for a system composed of a two-dimensional jet of glycerol aerosol and Desicote-coated glass microscope slides inclined to it. The patterns of particles deposited in these experiments were found to change with the velocity of the jet and the radius of the aerosol, in a manner different from previous assumptions that the particle would adhere to a solid surface at its first point of contact. Present tests indicate that a certain fraction of the particles bounces off from the surface on first contact. Authors discuss criterion droplets of adhering or bouncing off of impinging droplets on the basis of previous work of Gillespie, Rideal, and Rumpf, and define the concept of "sticking probability." Experimental set-up consisted of an aerosol generator, a flow conduit terminating in a rectangular nozzle, a machined plate containing the collecting surface, and intercepting devices for sampling purposes; these elements are illustrated and described. Procedure is explained in detail; some results of cumulative distribution of distances are shown in table and in chart. A qualitative explanation of the phenomena found is offered, based on the strain of the deformed droplet and its release, the viscous drag of the air, and the Brownian motion of the particle in the stream line. An equation expressing the adhesional force is offered; the effect of electrostatic force is slight compared with the van der Waals forces of attraction.

K. J. DeJuhasz, Germany

**5302. Endoh, K., and Oyama, Y., On the size of droplet disintegrated in liquid-liquid contacting mixer,** *Scient. Pap. Inst. Physical and Chemical Res.* **52**, 131–142 (Pap. 1486), Sept. 1958.

Rate of mass transfer to small particles suspended in a stirred tank of mixer-settler has a determining influence on the rate of extraction. Studies were made on the size of disintegrated droplets in liquid-liquid contacting mixers. Concept of power consumption per unit mass of agitated liquid is introduced, which is one of the factors of the principal velocity of the agitated liquid relative to the fully developed droplets. Equations are given for this principal velocity, in terms of average drop diameter, kinematic viscosity of continuous phase, and principal length of turbulence. An equation is given for the average drop size. In the experiments, mixing tanks of 14 and 20-cm diam were used, with 4- and 6-bladed paddle impellers. Different inmiscible liquids were used as dispersed phase and were mixed with about 99.8 vol. per cent of water. Each set of experimental data could be fitted to the same equation, by using different parameters.

K. J. DeJuhasz, Germany

**5303. Bergwerk, W., Flow pattern in diesel nozzle spray holes,** Instn. Mech. Engrs., Prepr., 6 pp., 1958.

Paper presents a study of the flow in spray holes of 0.2 to 2.5-mm diameter and shows how the changes in cavitation pattern affect the appearance of the jet. The influence of the cavitation number, Reynolds number, the upstream edge sharpness, and the length/diameter ratio is investigated.

A cavity first formed near the upstream corner, but soon caused the jet to leave the wall altogether so that only the upstream corner had any effect on the flow. Under noncavitating conditions the emerging jet had a ruffled appearance, but under conditions when the jet had left the wall, it emerged smooth and glass-like. The glass-like stage could only be obtained with very accurately made spray holes, and any disturbance upstream, such as occurs

in actual Diesel nozzles, caused the jet to appear ruffled at all times.

The discharge coefficient was found to vary with Reynolds number and cavitation number and a contour map covering Reynolds number of 1000 to 20,000 and cavitation number of 0.2 to 100 is presented. From author's summary

**5304. Lyshevskii, A. S., Determination of boundary velocities when liquid streams disintegrate** (in Russian), *Nauch. Trudi Novocherkaskii Politekhn. In-ta* **39**, 53, 67–70, 1957; *Ref. Zh. Mekh.* no. 4, 1958, Rev. 4075.

An empirical formula is given for the determination of the limits of different forms of the breaking-up of streams of water: (1) disintegration of the stream without reaction by atmospheric forces, (2) disintegration with such forces, (3) disintegration, with the formation of a wave outline. The boundaries between the forms of stream disintegration are determined by analysis of the results of experiments by O. Gol'fel'der [Process of disintegration of a stream in relation to the form of the jet and the counter pressure, Vol. 1. S. N. Vasili'ev, Editor, ONTI, NKTP, SSSR, 1936], with the application of the theory of dimensions; the equations of these boundaries have the form of

$$W = A\pi^m \qquad 5(a)$$

where $\pi$ is the relation of the density of liquid $\rho_l$ to the density of the air $\rho_a$

$$W = U_n^2 \rho_l d_c \alpha^{-1},$$

$U_n$ being the velocity of the stream, $d_c$ the diameter of the jets' outlet, $\alpha$ the surface tension of the liquid. For every boundary the numerical values of $A$ and $m$ were found. There are inaccuracies in the author's reasoning. He disregards the influence of the liquids' viscosity on the grounds that in the experiments the coefficient of viscosity remained a constant, but the same thing could be said in regard to the surface tension. Author holds the view that equation 5(a) is derived on the basis of the theory of similarity, whereas the theory of similarity only indicates that $W$ appears to be a function of $\pi$ and that the presentation of the relation of $W$ to $\pi$ in the form of a stepped function is only a permissible hypothesis. M. I. Gurevich

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5305. Degtev, O. N., Deformation of drops in a flow of gas and the drops' stability limit** (in Russian), *Trudi Ural'skogo Politekhn. In-ta*, no. 61, 106–112, 1956; *Ref. Zh. Mekh.* no. 4, 1958, Rev. 4080.

Paper is devoted to the investigation of the deformation and atomization of drops in a gas flow. Based on this the author is of the opinion that it is possible to differentiate the characteristic regimes in the flow of liquid jets (the presence or absence of secondary breakdown of drops after disintegration). It is assumed that the drop (when conditioned by a noticeable influence of the forces of surface tension) can be deformed either into a disk, or a cylinder with rounded ends. Computing the mean surface value of normal pressure on the sphere and levelling up its surface tension pressure, author obtains a value for the magnitude of the criterion of breakdown for the beginning of the deformation

$$D = \frac{\rho w^2 d}{\sigma} = 9.2 \qquad [1]$$

where $w$, $d$ are the relative velocity and diameter of the drop, respectivly, $\rho$ the gas density, $\sigma$ the coefficient of surface tension. Application of the analogous argument to the liquid cylinder (the relation of its length to the diameter is assumed to be 4) gives the critical value

$$D_* = \frac{\rho w_*^2 d}{\sigma} = 16.1 \qquad [2]$$

Here $w_*$ is the critical value of the relative velocity of the drop. Author proceeds to compare the figures obtained with the corresponding experimental values found by the abstractor and reports good agreement. The method employed by the author appears to be roughly approximate and is likely to yield only a conception of the order of values of the breakdown criterion.

M. S. Volynskii

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5306. Prokonenko, S. F., The geometrical features of the centrifugal nozzles of sprinklers** (in Russian), *Sel'khozmashina* no. 1, 17–18, 1957; *Ref. Zh. Mekh.* no. 4, 1958, Rev. 3988.

The problem is examined of the centrifugal atomizer into the vortex chamber of which the flow enters at an angle to the plane section $\alpha_{inlet} \neq 0$. It is shown that the end formulas for this case have the same form as the case of $\alpha_{inlet}$ being equal to zero if the value

$$A = \frac{R_{inlet} r_c \pi \cos \alpha_{inlet}}{nf_{inlet}}$$

is taken for the geometrical characteristic of the centrifugal nozzles of the sprinklers. Here $R_{inlet}$ is the distance from the longitudinal axis of the nozzles to the axis of the feeder channel, $r_c$ the radius of the outlet nozzle, $nf_{inlet}$ the total area of all the flow inlets into the vortex chamber.

V. I. Vasilév

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5307. Zrelov, N. P., Modelling of the settling of suspended particles** (in Russian), *Trudi Gidravl. Labor. Vses. N.-i. In-ta Vo-dosnab., Kanaliz., Gidrotekhn., Sooruzh. i Inzh. Gidrogeol.* no. 5, 79–109, 1957; *Ref. Zh. Mekh.* no. 4, 1958, Rev. 4311.

The various methods employed for modelling suspended particles are reviewed. The procedure employed for modelling is discussed in connection with the form of calculation reflection, proposed by the author, for the settlement of the particles in the quieting-down liquid. The modelling of the settling of sandy deposits is proposed to be done with the help of specially prepared balls of small density. The results of tests with these balls are given and practical recommendations are made regarding the application of the proposed method of modelling. B. A. Fidman

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5308. Bradley, D., and Pulling, D. J., Flow patterns in the hydraulic cyclone and their interpretation in terms of performance,** *Trans. Instn. Chem. Engrs.* **37**, 1, 34–45, Feb. 1959.

Dye injections into the fluid flowing in hydraulic cyclones have illustrated known features and have added to detailed knowledge of flow patterns. The results are described and the implications on theoretical correlation of efficiency are discussed. Such a correlation can never be precise owing to uncertainties in the path followed by an individual particle.

A theoretical approach is, however, given and is shown to provide results which compare favorably with performance data for a particular design of cyclone. From authors' summary

# Porous Media

(See also Revs. 4972, 5232)

**Book—5309. Carman, P. C., Flow of gases through porous media,** New York, Academic Press Inc., 1956, ix + 182 pp. $6.

Author has provided a concise monograph on gas flow through porous media which collects material previously available only in journals of widely diversified nature, and which will be valued highly by research workers. Emphasis is placed on the specific flow properties of a wide variety of consolidated and non-consolidated media, and no space is given to solutions of flow-field or boundary-value problems of the type encountered in petroleum technology, but reference is made to recent methods of treating nonlinear transient-flow problems. Subjects treated include not only flow of a single component under a pressure gradient, but diffusional flow along a partial pressure gradient in a gas mixture, and under a surface concentration gradient, when the gas is absorbed. Although the main treatment is nominally restricted to gas flows, the subject of permeability to two-phase flows is well summarized, and references to the literature are provided.

Treatment considers media with mean pore diameters ranging from the very small size, in which molecular slip is important, to the gross interstices of packed and fluidized beds. In discussing the modes of flow prevailing in the different media, author speaks in terms of transition from streamline to "turbulent" flow. Beginning with the two-term Reynolds relation between pressure gradient and flow velocity, $\Delta p/L = a\mu u + bpu^2$, author notes that $a$ is the inverse permeability coefficient for Darcy flow, but states that $b$ is not a true specific constant. Reviewer notes that the Reynolds approximation is strictly accurate only for fine-grained media, in which the pore size is too small to permit truly random, aperiodic, turbulent exchange of large groups of molecules. Inertial flow resistance has been observed in consolidated media of structure fine enough to show molecular slip effects at the low-flow end of the Darcy regime, and in such media the quadratic resistance is more probably attributable to molecular transfer of momentum in bound vortices distributed throughout the medium. In this case the local conditions at a given point are steady with respect to time, and the term "turbulent" is not appropriate. In the case of the coarser media, however, truly turbulent mixing within the pores is possible. In this case the two-term Reynolds expression does not fit the data well in the transition region, and addition of a third term is sometimes justified. Reviewer suggests that a forced fit of the two-term expression to transition-region data for coarse media may define $b$-values which could not be expected to be true specific constants.

L. Green, Jr., USA

**5310. Oroveanu, T., An approximate method for studying the flow toward oil wells in nonhomogeneous porous media** (in Roumanian), *Studii Si Cercetari Mecan. Appl.* **10**, 1, 87–108, 1959.

Author considers the equation of flow in a nonhomogeneous porous medium under the form

$$\Delta\varphi + \frac{\partial(\log k)}{\partial x}\frac{\partial\varphi}{\partial x} + \frac{\partial(\log k)}{\partial y}\frac{\partial\varphi}{\partial y} = 0$$

where $\varphi = -\frac{1}{\mu}(p + U)$, $U$ being the force function and $k$ the slowly varying permeability. By setting $\frac{\partial(\log k)}{\partial x} = l$, $\frac{\partial(\log k)}{\partial y} = n$ and $\varphi = \Phi e^{-\frac{1}{2}(lx + ny)}$ he obtains

$$\Delta\Phi - \alpha^2\Phi = 0$$

A first solution is determined in a rim with given constant pressures on both its circular contours. Another solution refers to an oil well considered as a punctiform source placed at a point within the circular domain. Formulas for the pressure and rate of flow are obtained with corresponding boundary conditions. An extension is included for the case of $n$ sources placed arbitrarily within the circular domain. St. N. Savulescu, Roumania

**5311. Fil'chakov, P. F., Graphic analytical method for calculating the filtration discharge of spillways** (in Russian), *Gidrotekhn. Stroit.* no. 10, 43–50, 1956; *Ref. Zh. Mekh.* no. 4, 1958, Rev. 4324.

N. N. Pavlovskii's problem is first examined; this deals with the flow of a single channel spillway with an infinite depth for the bed of the apron. The conforming conversion of a single channel spillway in a plane $z$ to a supplementary semiplane $\zeta^*$, accomplishable by the function

$$\zeta^* = \sqrt{S^2 + Z^2},$$

is carried out graphically by means of compass and rule; subsequently all the characteristics of the filtration flow are determined. Nomograms are furnished from which values can be read off for the adduced pressure and the adduced discharge, passing through some portion of the "lower water." A description is then given of the graphical calculation for a multichannel, not deeply set and deeply set, spillway. Results of the calculations by the proposed method are compared with the results obtained by the analytical method and also with those obtained by the Blei and Len methods. A numerical comparison shows that the precision of the proposed method is amply sufficient for practical computations. Author indicates that the graphoanalytical method permits the carrying-out in an ocular form of comparisons of the work of various spillways, and gives an example of such a comparison. In conclusion a calculation is demonstrated for spillways of practical profile and it is shown that the graphoanalytical method can be applied even for a finite power of the water permeable layer $T$, provided that the condition $T > 2S$ can be fulfilled, where $S$ is the length of the longest channel. If this condition can not be met, then the alternative proposal is made to use the method of successive conforming reflections worked out by the author.

V. A. Vasilév

*Courtesy Referativnyi Zhurnal, USSR*

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**5312. Guzov, M. Z., Filtration calculation for earth dams taking into account the water permeability of the protective shielding and front of the spillway** (in Russian), *Trudi Kievsk. Gidromelior. Inta* no. 6, 115–118, 1956; *Ref. Zh. Mekh.* no. 4, 1958, Rev. 4326.

This is a comparatively simple hydraulic method for calculating the filtration through earth dams having water permeable shields and spillway fronts. By determining separately the filtration discharges passing through the shield, spillway front and permeable foundation and by comparing the sum of these discharges with the filtration discharge passing below the dam, determined by the Dupuis equation, author obtains a quadratic equation relating to the magnitude of the pressure below the end part of the front of the spillway. Author continues by obtaining a simple expression for the equation of the depression curve which contains the given value. The proposed method is simpler than known methods for calculating the filtration discharge of analogous installations. It should be noted that for the purpose of evaluating the accuracy of his method author utilizes data of experiments at the Akulovsk dam [P. A. Shankin, "Calculations for filtration in earth dams." Moscow, Izd-vo M-va Rechn. Flota, 1947] and asserts that his method gives the more proximate results in comparison with the experiment (divergence 2.5%), and with additional simplicity. However in work done by the abstractor ["Calculation of filtration discharge through earth dams." Leningrad-Moscow, Stroiizdat, 1940] it was shown that the model of the Akulovsk dam was equipped with a spur 5.0 m deep; this, with a thickness for the permeable foundation of 13 m, would form a definite factor in any supplementary damping of the pressure. Consequently theoretical

methods are bound to give definitely higher values for the ordinates in comparison with the experimental values, obtained in the face of supplementary resistance due to a spur. Therefore the author's method, giving a reduction in ordinate values of 2.5%, does not appear to be sufficiently accurate. The comparative simplicity in carrying out the calculations appears to be its most positive favorable quality. It should also be noted that the author, in borrowing P. A. Shankin's Table I, reproduces results containing faulty calculations. A. A. Uginchus

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An approximate hydraulic method is given for the calculations for filtration of earth dams, consisting of a number of sloping layers placed parallel with differing filtration coefficients. The assumption is made that in the first layer the lines of current are normal to the slope and that in subsequent layers they are parallel to the impermeable foundation of the dam. By applying d'Arcy's principle to each layer successively, author obtains a system of semi-filtration equations with four unknowns. These equations are recommended by the author for selective use, which is illustrated by a numerical example. The filtration calculation for the lower wedge section, that is through the last layer of the dam, is recommended to be carried out by formulas known in the literature but now somewhat obsolescent (P.A. Shankin or L. Kazagrande). The assumptions contained in the proposed method can only be evaluated after an experimental check, for instance on the EGDA apparatus. The arguments put forward for rejection of "the virtual" method of N. N. Pavlovskii, referred to in the work of P. A. Shankin ["Calculations for filtration in earth dams." Moscow, Izd-vo M-va Rechn. Flota, 1947], are not convincing. A. A. Uginchus

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**5314. Kirkham, D., Seepage of steady rainfall through soil into drains,** *Trans. Amer. Geophys. Un.* **39**, 5, 892–908, Oct. 1958.

Paper studies theoretical two-dimensional flow of ground water into regularly spaced circular or vertical drains in soil underlain by impermeable layer. Owing to mathematical difficulty of using true free boundary (ground-water table), analyses are based on horizontal upper boundary with constant flow/unit width. Although assumption is unrealistic, author obtains good agreement with some field data on Dutch and tile drains. For deep (slit) vertical drains, it is further assumed that seepage rate into slit is constant over slit height. Latter assumption is also needed for mathematical analysis. Table for practical computations is included.

Value of paper would be enhanced by inclusion of some comparison with analog results (e.g. conducting paper technique) for correct boundary conditions. T. K. Chaplin, England

**5315. Kolbuszewski, J., Stewart, J. C., and Hu, G. C.-Y., A comparative study of experimental techniques for flow net determinations. Parts I and II,** *Civ. Engng., Lond.* **53**, 626, 901–903, Aug. 1958; **53**, 627, 1026–1029, Sept. 1958.

Authors compare trial solutions of four representative seepage problems obtained by three experimental methods: (1) sand model in a water tank; (2) electrical analogy on conducting paper; (3) electrical analogy on a passive resistance network. The sand model requires careful technique, and involves errors due to nonuniform and nonisotropic permeability distribution and to capillary rise above the seepage line, but is recommended for purposes of flow visualization and instruction. The resistance network yields the most accurate results (as established by comparison with relaxation solutions), provided that the mesh size is small enough or the boundary geometry simple enough that the boundary conditions can be well approximated. The conducting paper represents a good compromise between the other techniques, permitting complex boundaries to be handled easily and yielding intermediate accuracy. Authors' treatment might have benefited by more thorough reference to investigations reviewed by Higgins, AMR **9**, p. 1, 1956; AMR **9**, p. 49, 1956; AMR **10**, p. 49, 1957.

L. Green, Jr., USA

**5316. Gibson, R. E., The progress of consolidation in a clay layer increasing in thickness with time,** *Géotechnique, Lond.* **8**, 4, 171–182, Dec. 1958.

A clear exposition of the estimation of the increase in pore pressure in a clay mass during the construction of a dam or fill is presented. The mathematics is rigorous and the assumptions, though oversimplified, are rational. Graphs permit the estimation of the pore pressure as a function of the increase in thickness of the fill with time. When used with pore-pressure measurements during construction, comparison of calculated and observed data makes it possible to adjust the design and rate of building to maintain the factor of safety.

K. N. Hendrickson, USA

## Geophysics, Hydrology, Oceanography, Meteorology

**Book—5317. Duke, M. C., Bibliography of effects of soil conditions on earthquake damage,** San Francisco, Earthquake Engineering Research Institute, 1958, iii + 47 pp. $1. (Paperbound)

**5318. Knopoff, L., Surface motions of a thick plate,** *J. Appl. Phys.* **29**, 4, 661–670, Apr. 1958.

One of the serious difficulties of experimental and theoretical seismology arises from the complexity of the motions produced by a pressure pulse located at the surface of an elastic half-space (Lamb) or in the interior of it (Lapwood). For the case of a solid bounded by two parallel planes matters are even more complex. Author derives closed form solution for the motion of the epicenter. For pulses whose duration is short compared to the travel times of P and S waves through the thickness of the plate the significant features of the motion are found to be given by the behavior of the response function at the arrival times. From this observation author shows how a simplified description of the response, a compromise between the exact solution of the mathematical theory of elasticity and the elementary arrival time predictions of geometrical seismology, may be constructed. Interesting measurements of the response at points on the base of a limestone plate are presented in support of the analysis. Away from the epicenter the S pulse attains an amplitude approximately four times that of the P pulse. A bodily S-surface P phase is observed at angles exceeding the critical angle.

L. E. Goodman, USA

**5319. Rossiter, J. R., On the application of relaxation methods to oceanic tides,** *Proc. Roy. Soc. Lond.* (*A*) **248**, 1255, 482–498, Dec. 1958.

Author demonstrates the use of relaxation methods for the solution of forced tides by application to a hypothetical ocean basin formed by two meridians and two parallels of latitude symmetrical about the equator. The efficacy and accuracy of the procedure are illustrated first by application to a simple one-dimensional problem (forced tide in a meridianal canal) for which the

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V. A. Vasilév

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*Courtesy Referativnyi Zhurnal, USSR*
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**5314. Kirkham, D., Seepage of steady rainfall through soil into drains,** *Trans. Amer. Geophys. Un.* **39**, 5, 892–908, Oct. 1958.

Paper studies theoretical two-dimensional flow of ground water into regularly spaced circular or vertical drains in soil underlain by impermeable layer. Owing to mathematical difficulty of using true free boundary (ground-water table), analyses are based on horizontal upper boundary with constant flow/unit width. Although assumption is unrealistic, author obtains good agreement with some field data on Dutch and tile drains. For deep (slit) vertical drains, it is further assumed that seepage rate into slit is constant over slit height. Latter assumption is also needed for mathematical analysis. Table for practical computations is included.

Value of paper would be enhanced by inclusion of some comparison with analog results (e.g. conducting paper technique) for correct boundary conditions. T. K. Chaplin, England

**5315. Kolbuszewski, J., Stewart, J. C., and Hu, G. C.-Y., A comparative study of experimental techniques for flow net determinations. Parts I and II,** *Civ. Engng., Lond.* **53**, 626, 901–903, Aug. 1958; **53**, 627, 1026–1029, Sept. 1958.

Authors compare trial solutions of four representative seepage problems obtained by three experimental methods: (1) sand model in a water tank; (2) electrical analogy on conducting paper; (3) electrical analogy on a passive resistance network. The sand model requires careful technique, and involves errors due to nonuniform and nonisotropic permeability distribution and to capillary rise above the seepage line, but is recommended for purposes of flow visualization and instruction. The resistance network yields the most accurate results (as established by comparison with relaxation solutions), provided that the mesh size is small enough or the boundary geometry simple enough that the boundary conditions can be well approximated. The conducting paper represents a good compromise between the other techniques, permitting complex boundaries to be handled easily and yielding intermediate accuracy. Authors' treatment might have benefited by more thorough reference to investigations reviewed by Higgins, AMR **9**, p. 1, 1956; AMR **9**, p. 49, 1956; AMR **10**, p. 49, 1957.

L. Green, Jr., USA

**5316. Gibson, R. E., The progress of consolidation in a clay layer increasing in thickness with time,** *Géotechnique, Lond.* **8**, 4, 171–182, Dec. 1958.

A clear exposition of the estimation of the increase in pore pressure in a clay mass during the construction of a dam or fill is presented. The mathematics is rigorous and the assumptions, though oversimplified, are rational. Graphs permit the estimation of the pore pressure as a function of the increase in thickness of the fill with time. When used with pore-pressure measurements during construction, comparison of calculated and observed data makes it possible to adjust the design and rate of building to maintain the factor of safety.

K. N. Hendrickson, USA

## Geophysics, Hydrology, Oceanography, Meteorology

**Book—5317. Duke, M. C., Bibliography of effects of soil conditions on earthquake damage,** San Francisco, Earthquake Engineering Research Institute, 1958, iii + 47 pp. $1. (Paperbound)

**5318. Knopoff, L., Surface motions of a thick plate,** *J. Appl. Phys.* **29**, 4, 661–670, Apr. 1958.

One of the serious difficulties of experimental and theoretical seismology arises from the complexity of the motions produced by a pressure pulse located at the surface of an elastic half-space (Lamb) or in the interior of it (Lapwood). For the case of a solid bounded by two parallel planes matters are even more complex. Author derives closed form solution for the motion of the epicenter. For pulses whose duration is short compared to the travel times of P and S waves through the thickness of the plate the significant features of the motion are found to be given by the behavior of the response function at the arrival times. From this observation author shows how a simplified description of the response, a compromise between the exact solution of the mathematical theory of elasticity and the elementary arrival time predictions of geometrical seismology, may be constructed. Interesting measurements of the response at points on the base of a limestone plate are presented in support of the analysis. Away from the epicenter the S pulse attains an amplitude approximately four times that of the P pulse. A bodily S-surface P phase is observed at angles exceeding the critical angle.

L. E. Goodman, USA

**5319. Rossiter, J. R., On the application of relaxation methods to oceanic tides,** *Proc. Roy. Soc. Lond.* **(A) 248**, 1255, 482–498, Dec. 1958.

Author demonstrates the use of relaxation methods for the solution of forced tides by application to a hypothetical ocean basin formed by two meridians and two parallels of latitude symmetrical about the equator. The efficacy and accuracy of the procedure are illustrated first by application to a simple one-dimensional problem (forced tide in a meridianal canal) for which the

exact solution is known. An ingenious transformation of the colatitude coordinate is employed in order to enhance the convergence properties of the procedure. The finite difference equations and boundary conditions required in the solution of the two-dimensional tide problem are described clearly and the features of the relaxation procedure unique to the tide problem are described in some detail. Application is made to a basin having roughly the dimensions of the Atlantic ocean for the $K_2$ tide and results are compared with those obtained by Doodson for the case of a basin bounded only by two meridians (i.e., terminating at poles). The presence of the boundaries formed by the parallels of latitude in the author's numerical solution causes a marked shift in the amphidromic point away from the central meridian of the basin. R. O. Reid, USA

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From authors' summary

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## Friction, Lubrication and Wear

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Authors feel that it may be of greater practical significance to evaluate qualitatively the changes that take place in the frictional characteristics (type as well as magnitude) with given changes in the above-described variables. The observed behavior may then be related to performance in the various unit processes employed in textile manufacturing. Evidence justifying this point of view is presented in the paper, and some rather speculative explanations correlating finish properties and frictional behavior are given. From authors' summary

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oil is used as fuel. It is also shown that in principle it is possible to determine the relationship between lubricant, fuel and additives using engine tests which in turn must form the foundation for basic and applied research into the chemistry of these materials. K. Schneider, USA

**4340. Constantinescu, V. N., On the motion stability of gas-lubricated journal bearings** (in Roumanian), *Studii Si Cercetari Mecan. Appl.* **10**, 1, 117–140, 1959.

Author considers the pressure equation for gas lubrication in the isothermic two-dimensional unsteady case, the time entering in a first approximation as a parameter through the angular velocity of the line of centers. The influence of this rotation on the pressure resultant is manifested through the multiplication of the latter by the factor $1 + \frac{2\dot{\varphi}}{\Omega_1 + \Omega_2}$, where $\dot{\varphi}$ is the angular rotation velocity of the line of centers, $\Omega_1$ is the angular rotation velocity of the shaft and $\Omega_2$ is the angular rotation velocity of the bushing. The equations of motion stability are written in compliance with N. Tipei's small perturbation method. A compatibility equation is obtained under the form of a fourth-order equation which is analyzed with the aid of Routh-Hurwitz conditions. By giving the magnitude and the direction of the load for the basic undisturbed regime as well as the kinematic and geometric elements of the motion, one may calculate their variation with respect to the relative eccentricity and the rotation of the line of centers, and hence the motion stability can be estimated.

Calculations are performed for steady motion and centrifugal loading. On discussing the obtained values, author concludes that generally the motion stability of air bearings is not ensured. As distinguished from the case of liquid lubrication, it seems that the eccentricity no longer plays an essential part in the demarcation of the instability zones. The physical explanation of the pronounced instability in the case of gas lubrication lies in the possibility of depression zones being formed due to gas compressibility and expansibility. St. N. Savulescu, Roumania

**5341. Rieger, N. F., An experimental investigation of the pressurized air-lubricated journal bearing in the high-speed range,** Instn. Mech. Engrs., Prepr., 10 pp., 1958.

**5342. Cameron, A., The viscosity wedge,** *ASLE Trans.* **1**, 2, 248–253, Oct. 1958.

A thrust bearing consisting of a stationary step and moving collar having parallel surfaces was investigated. The flow of lubricant through the bearing, its load-carrying capacity as a function of film thickness, and coefficient of friction were analyzed.

The following assumptions were made: (1) the temperature of the lower (stationary) surface is constant ($T_0$), (2) the temperature of the upper (moving) surface is the same as that of lower surface ($T_0$) only at the inlet edge of the two surfaces and increases uniformly in the direction of motion, so that the temperature becomes ($T_0 + t$) at the outlet edge of the upper surface, and (3) the temperature difference is linear through the thickness of the oil film (i.e. the temperature gradient is constant across the film).

An "infinite pad" (bearing with side leakage neglected) was considered in this investigation.

Equations for the load capacity as a function of film thickness and for coefficient of friction were developed and solved numerically.

Author concludes that, under the assumed conditions, the variation of the oil viscosity across the oil film ("viscosity wedge") may have an effect on the load capacity of a thrust bearing of the same order of magnitude as the effect of "Michell Wedge" (inclination in a converging oil film), and that the variation of the oil viscosity across the oil film has much greater effect on bearing capacity than the expansion of the oil due to progressive heating up of the oil in its passage through the bearing (so-called "thermal wedge"). This analysis showed also that, under the assumed conditions, the coefficient of friction is greatly influenced by the total temperature rise in the oil film $t$. E. I. Radzimovsky, USA

**5343. Cameron, A., and Wood, W. L., Parallel surface thrust bearing,** *ASLE Trans.* **1**, 2, 254–265, Oct. 1958.

Fogg's thermal wedge effect for parallel surface thrust bearing is analyzed theoretically, assuming all heat removed by oil and viscosity constant across oil film. Basic equations of continuity and energy are set up in dimensionless form, an exponential relationship between viscosity and temperature is introduced, solutions are obtained and graphed for the infinitely wide bearing pad. Solutions show that the temperature rise is much greater than that for the tilting pad bearing and the load capacity is much less. Comparison of results for constant and variable viscosity shows that taking a constant value of viscosity corresponding to that at two-thirds of total temperature rise gave a very good approximation for load and friction coefficient. Solutions are obtained for finite pad using relaxation methods, assuming constant viscosity, in terms of Bessel and Struve functions.

This paper originated in 1946 and should be used in conjunction with "The viscosity wedge." *ASLE Trans.* **1**, no. 2, p. 248 (see preceding review). C. F. Kettleborough, New Zealand

**5344. Otterbein, M. E., The effect of aircraft gas turbine oils on roller bearing fatigue life,** *ASLE Trans.* **1**, 1, 33–39, Apr. 1958.

**5345. Pavlenko, A. V., Determination of the deformation of a plastic slip bearing due to the action of hydrodynamic pressure** (in Russian), *Trudi Khar'kovsk. Politekhn. In-ta* **10**, 3, 107–114, 1956; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 2977.

On the surface of a plastic bearing, in the zone of the maximum hydrodynamic pressures, a hole is formed, as shown by the tests, which assists in increasing to some extent the minimum thickness of the lubricant's film by comparison with that obtained when using a metallic bush. From author's summary

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5346. Sharapin, E. F., Some principles of the anisotropic friction of slip** (in Russian), *Trudi Khar'kovsk. Politekhn. In-ta* **9**, 1, 29–39, 1957; *Ref. Zh. Mekh.* no. 4, 1958, Rev. 3679.

A concept is advanced regarding "the curve of the friction coefficient" in polar coordinates $f = F(\alpha)$ [1], where $f$ is the friction coefficient in the direction characteristic of the angle $\alpha$. If the curve [1] has two mutually perpendicular axes of symmetry, the friction is termed orthotropic. In the general case the direction of the relative slip of the frictional (rubbing) pair makes up angle $\delta$ with the direction of the shear force and can be found by minimizing the specific work done by friction, leading to the equation $tg = \pm(F'(\alpha)/F(\alpha))$. The components of friction force $T$ in the direction of slip and normal to it are equal to $T \cos \sigma$ and $T \sin \sigma$, respectively. The values of $\alpha$ when $\delta = 0$ determine the principal directions of slip. In the absence of special points in curve [1], the principal directions coincide with the axes of symmetry; then zones (sectors) will appear of reduced or increased stability of slip, containing the given axes. A method is described for the experimental delineation of curves for the coefficient of friction.

B. M. Broude

*Courtesy Referativnyi Zhurnal, USSR*
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**5347. Sorensen, S. V., and Slutskaya, O. B., Strength of crankshafts of tractors considered in conjunction with wear of the bearings** (in Russian), Increase of wear-resistance and life of machines, Kiev-Moscow, Mashgiz, 1956, 183–190; *Ref. Zh. Mekh.* no. 3, 1958, Rev. 3484.

Statistical data are used to confirm the sound basis of the process of changes in the degree of stress and the appearance of fatigue stresses in the crank shafts of tractors in consequence of wear. This explains the fracture of the shafts after 50 million or more cycles of stress change. The values are calculated, using the customary methods of structural mechanics, of the relative support displacements and the magnitude of the supplementary forces appearing in consequence of the lack of co-axiality in the bearings. The forces which operate on the crankshaft with unworn bearings and with bearings which are not co-axial are determined experimentally. It was determined that for the shaft of a tractor of type D-54 the limit of lack of co-axiality amounts to 0.1 – 0.15 mm. The elasticity of the gearbox appears to have no influence on the stability of the shafts. It is explained that the fundamental reason for the fatigue-fracture of shafts with large reserves of worked hours is the uneven wear of the main bearings. There is here a fresh distribution of forces, in consequence of redundancy, leading to the overloading of a number of sections and a sharp drop in the stability-safety factor. A similar influence on the reliability of the shaft would point to faulty manufacture and incorrect assembly.

V. K. Pereverzev

*Courtesy Referativnyi Zhurnal, USSR*
*Translation, courtesy Ministry of Supply, England*

**5348. Anderson, W. J., and Carter, T. L., Effect of lubricant viscosity and type on ball fatigue life,** *ASLE Trans.* **1**, 2, 266–272, Oct. 1958.

**5349. Davidson, T. F., and Ku, P. M., The effect of lubricants on gear tooth surface fatigue,** *ASLE Trans.* **1**, 1, 40–50, Apr. 1958.

This paper presents some of the results of a recent investigation on the effect of lubricants on gear tooth surface fatigue (pitting). The work described comprises the development of a laboratory test procedure for exploratory purposes, and the determination of the performance of a number of lubricants using this procedure. It was found that bulk lubricant viscosity, as measured by the conventional viscometric method, did not have a predominant effect on gear tooth surface fatigue. Examination of the development of scuffing and pitting on the gear teeth indicates that the initial incidence of scuffing and the initial incidence of pitting were unrelated. It was further noted that as pitting became advanced, severe scuffing of the pitted teeth would almost invariably result. On the other hand, there was no evidence that severe scuffing would lead to gear tooth pitting.

From authors' summary

## Letters to the Editor

**5350.** Re: AMR **12** (May 1959), Rev. 2417: **Benthem, J. P., The enclosure of the flexibilities, the cross-flexibility included, of infinite hollow box beams with oblique ribs** (in English), 9th Congrès Intern. Mécan. Appl., Univ. Bruxelles, 1957; **6**, 160–167.

The reviewer supposes an error in computing the effective spar boom area. However, if a set of discrete stringers on the skin is continuized properly, each spar boom area is to be diminished with half a stringer area. This procedure was carried out properly.

The solution of Hemp must lie outside the given bounds for theoretical reasons.

J. P. Benthem, Holland

**5351.** Re: AMR **12** (June 1959), Rev. 2888: **Henri, M., Virtual stresses and the surface of a weak sliding resistance inside a massive body** (in French), *Ann. Inst. Tech. Bat. Trav. Publics* no. 120, 1384–1402, Dec. 1957.

Author's name should read Marcus, H., instead of Henri, M., in the review and also in the authors' index. The editors regret this error.

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AHLREN, F., Den termodynamiska svangningsteorien, Goteborg, Sweden, Gumperts, 1958, 103 pp. kr. 15 (Paperbound).

CONDOYANNIS, G. E., Scientific Russian, New York, John Wiley & Sons, Inc., 1959, xii + 225 pp. $3.50.

DUSHEK, A., Hohere Mathematik Vol. II, Integration und Differentiation der Funktionen von mehreren Veranderlichen. Lineare Algebra. Tensorfelder Differentialgeometrie, second edition, Wien, Springer-Verlag, 1958, viii + 401 pp. $11.45.

GREEN, H. L., and LANE, W. R., Particulate clouds: dusts, smokes and mists; their physics and physical chemistry and industrial and mists, Princeton, N. J., D. Van Nostrand Co., Inc., 1957, xix + 425 pp. $11.25.

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KAROLYI, Z., and STAROSOLSZKY, O., Az ontozoviz merese II, Tanulmanyok es kutatasi eredmenyek no. 5, Budapest, Vizgazdalkodasi Tudomanyos Kutato Intezet, 1958.

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NIEBERGALL, W., Sorptions-Kaltemaschinen; Vol. 7, Handbuch der Kaltetechnik edited by R. Plank, Berlin, Springer-Verlag, xviii + 540 pp. DM 76.80.

ORR, C., JR., and DALLA VALLE, J. M., Fine particle measurement size, surface and pore volume, New York, The Macmillan Co., 1959, xiv + 353 pp. $10.50.

PRIESTLEY, C. H. B., Turbulent transfer in the lower atmosphere, Chicago, U. of Chicago Press, 1959, vii + 130 pp. $3.75.

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SAUER, R., Anfangswertprobleme bei partiellen Differentialgleichungen, second edition, Berlin, Springer-Verlag, 1958, xv + 284 pp. DM 41.

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Seventh Symposium International on Combustion, London and Oxford 28 August-3 September, 1958, New York, Academic Press, Inc., 1959, xlvi + 959 pp. $35.

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